AFAL-TR-78-16
Volume I

1

AD-A051885

FIRE CONTROL SYSTEM ANALYSIS

Volume I - Analysis Tasks

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November 1977

Technical Report AFAL-TR-78-16, Vol I

Final Report for Period 1 Nov 76 - 30 Sep 77

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFAL-TR-78-16, Volume I	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Fire Control System Analysis Volume I - Analysis Tasks		5. TYPE OF REPORT & PERIOD COVERED Final Report 11/1/76-9/30/77
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Lowell Ryan, Ron Reboulet, Steven Gustafson Philip Fry		8. CONTRACT OR GRANT NUMBER(s) F33615-77-C-1056
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Dayton Research Institute Dayton, Ohio 45469		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 63605F/76290359
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Avionics Laboratory Wright-Patterson AFB, Ohio 45433		12. REPORT DATE November 1977
		13. NUMBER OF PAGES 214
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Kalman Filter, Target State Measurement, Radar Lag, Multiple Digital Scan Converter, Air Combat Evaluation Algorithm

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The following three tasks are included in Volume I. Gaussian noise added to true range generated by various kinds of simulated trajectories served as "measurements" which were input to several kinds of Kalman filters. Filter output is compared graphically and by rms deviations. Independent methods were devised for evaluating the accuracy of target state vectors (position, velocity, and acceleration) obtained from a director fire control system.
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20. ABSTRACT (Continued)

Attempts to find an adequate software correction to the radar lag were not successful. Analysis of data on Sight Eval tapes casts doubt on its validity.
Volume II contains reports on the following tasks. The equations used in the LCOS, TRACER, and ACE algorithms were rewritten such that they could be performed on the MDSC computer. The ACE algorithm was modified for implementation on the ROLM 16/64 computer.

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SECTION 1
BACKGROUND

The recognition by the Air Force Avionics Laboratory (AFAL) of the need for improved sensors and techniques for implementing fire control director algorithms has resulted largely from the conclusions of two programs - EXPO V and Fire/Fly. EXPO V is the latest in a series of man-in-the-loop simulation studies of new fire control and gunsight concepts. The Fire/Fly program is the integrated Fire Control/Flight Control study which is investigating methods for integrating the fire control system with the flight control system to improve effectiveness while increasing survivability. Both programs have cited the necessity for improved sensors as well as improved director algorithms.

The purpose of the Fire Control Systems analysis effort, summarized in the following report, was to examine existing fire control systems test data (Sight Eval), identify error sources, evaluate and modify fire control algorithms, and perform programming for simulation, weapon system investigation, and validation.

The tasks contained in this volume were of an analytical nature.

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SECTION 2
TASK DEFINITION

The broad categories of effort contained in the Statement of Work were refined by coordination with Captain J. Silverthorn, RWT-2, and resulted in the following series of tasks.

2.1 IDENTIFY RADAR RANGE ERRORS AND DEVELOP FILTER

An important input to the director gunsight computations is target range. As a result, it is desirable to have as accurate a range measurement as possible. Some type of filter (Kalman or other) is needed to produce an accurate estimate of range (and range rate). To accomplish this task, an approximate noise model is needed to aid in the filter design process. This noise model can also be used in the Air Force Flight Dynamics Laboratory (AFFDL) LAMARS simulation.

2.2 TARGET STATE MEASUREMENT

Two important subsystems of the director system are the target position sensor and Kalman angle filter. An appropriate performance measure for these subsytems is their ability to estimate target state (position, velocity, acceleration). As a result, to evaluate their performance it is necessary to independently measure these target parameters more accurately than the expected estimation errors. A technique must be developed and analyzed to accomplish this task.

2.3 IDENTIFY AND CORRECT RADAR LAG PROBLEM

The MA-1 radar was seen to lag the target by as much as 50 mrad during the Sight Eval flight test. Since this same radar is to be used during the director flight test for initial acquisition and handoff to the ASCOT sensor, it is necessary to be able to predict the lag so that the net error (difference between target true position and radar estimate) is less than ASCOT scan size ($\pm$ 15 mrad).

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SECTION 3
ANALYSIS SUPPORT

3.1 IDENTIFY RADAR RANGE ERRORS AND DEVELOP FILTER

3.1.1 Introduction

This report evaluates the performance of certain optimal and sub-optimal Kalman filters in yielding estimates of target state variables. The evaluation uses simulated trajectories with noise added to the true range as input to the filters, and compares filter estimates of the state variables to their true values.

A Kalman filter is often designed with respect to the coordinate system in which the target position is measured; normally the inertial or line-of-sight coordinate system. The filters considered in this report use only range information. Therefore, to give a complete specification of the target state, the relative range, relative radial velocity, and relative radial acceleration estimates yielded by these filters must be converted to a useful frame by coordinate transformations based on angular position, angular rate, and angular accelerations as measured by suitable devices and estimated by other filters.

Four Kalman range filters were compared. The first three were based on a constant relative radial acceleration model and the fourth on a constant relative radial velocity model.

To evaluate and compare these filters, trajectories of various types were generated, noise was added to each true range, and the resulting "measurement" was used as input to each filter.

Results are given graphically and as root-mean-square deviations of the filter estimate of range, (radial) velocity, and (radial) acceleration over a chosen interval.

The simulated constant acceleration trajectories assume (constant) accelerations of 0, 1g (9.8 m s^{-2}), and 6g (58.8 m s^{-2}). Two simulations assume a constant negative acceleration. The initial range and velocity were assumed to be 600 m and 200 m s^{-1}, respectively.

Two similar trajectories resulting from variable acceleration were also used as simulations. These resulted from sinusoidal accelerations of 8 sec. period and amplitudes of either 2g or 6g. The initial range was assumed to be 600 m and the initial velocity was zero.

3.1.2 Theory

Filter A: Constant Acceleration Model - Time Varying Gain

Let the state vector representing the target be denoted by

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

where $x_1(t)$, $x_2(t)$, and $x_3(t)$ represent the range, (radial) velocity, and (radial) acceleration, respectively, of the target at time t. The constant acceleration model assumes that the state vector evolves in time according to

$$\dot{x}_1=x_2$$
$$\dot{x}_2=x_3$$
$$\dot{x}_3=w(t)$$

where w(t) is assumed to be zero-mean, Gaussian noise uncorrelated with itself in time. This model can be written in matrix form as

$$\dot{\underline{x}}(t) = F\underline{x}(t) + Gw(t) \tag{1}$$

where

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Assuming that $w(t)$ is constant over the interval (t_o,t), the solution of Eq. (1) is given by

$$\underline{x}(t) = \phi(t,t_o)\underline{x}(t_o) + \left[\int_{t_o}^{t} \phi(t,t\tau)G(\tau)d\tau\right]w(t) \tag{2}$$

where

$$\phi(t,\tau) = \begin{bmatrix} 1 & t-\tau & \frac{1}{2}(t-\tau)^2 \\ 0 & 1 & t-\tau \\ 0 & 0 & 1 \end{bmatrix}$$

For successive times t_{k+1} and t_k, Eq. (2) becomes

$$\underline{x}(t_{k+1})=\phi(t_{k+1}, t_k)\underline{x}(t_k) + \left[\int_{t_k}^{t_{k+1}} \phi(t_{k+1},\tau)G(\tau)d\tau\right]w(t_k) \tag{3}$$

Defining

$$\underline{x}(t_{k+1}) \equiv \underline{x}(k+1)$$

$$\underline{x}(t_k) \equiv \underline{x}(k)$$

$$w(t_k) \equiv w(k)$$

$$\int_{t_k}^{t_{k+1}} \phi(t_{k+1},\tau)G(\tau)d\tau \equiv \Gamma(k+1,k),$$

Equation (3) can be written as

$$\underline{x}(k+1)=\phi(k+1,k)\underline{x}(k) + \Gamma(k+1,k)w(k)$$

By direct integration,

$$\Gamma(k+1),k) = \int_{t_k}^{t_{k+1}} \phi(t_{k+1},\tau)G(\tau)d\tau = \begin{bmatrix} \frac{1}{6}T^3 \\ \frac{1}{2}T^2 \\ T \end{bmatrix},$$

where

$$T = t_{k+1} - t_k$$

and $$\phi(k+1,k) = \phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$

Both T and ϕ are constant given a fixed interval between successive measurements.

It is assumed that the maneuver noise term, w(k), is zero-mean Guassian noise with the autocorrelation property

$$E\{w(j)w'(k)\} = Q(k)\delta_{jk} \tag{4}$$

where

$$Q(k)=\sigma_w^2 \qquad \text{(maneuver-noise variance)}$$

and δ_{jk} is the Kronecker delta.

A measurement model

$$z(k+1) = H\underline{x}(k+1) + v(k+1) \tag{5}$$

is assumed where v(k+1) is the (scalar) measurement noise,

$$H=[1 \quad 0 \quad 0]$$

is the measurement matrix, and z(k+1) is the measurement at time t_{k+1}. Equation (5) may be written in the scalar form

$$z(k+1) = x_1(k+1) + v(k+1) \tag{6}$$

It is assumed that the measurement noise has the properties

$$E\{v(j)v(k)\} = R(k)\delta_{jk} \tag{7}$$

and

$$E\{v(j)w(k)\} = 0$$

where

$$R(k) = \sigma_v^2$$

is the measurement noise variance; also called the signal noise variance.

The "best" estimate of the state vector at time t_k is denoted by $\hat{\underline{x}}(k|k)$; this estimate based on k measurements z(k), z(k-1), ..., z(2), z(1). The covariance matrix P(j|k) is defined to be

$$P(j|k)=E\left\{[\underline{x}(j)-\hat{\underline{x}}(j|j)][\underline{x}(k)-\hat{\underline{x}}(k|k)]'\right\} ,$$

where the transpose is indicated by a prime.

With these definitions, the Kalman equations are, in the logical order of their execution

$$\left.\begin{aligned}
&P(k+1|k)=\phi P(k|k)\phi'+\Gamma Q(k)\Gamma' && \text{(i)}\\
&\underline{K}(k+1)=P(k+1|k)H'[HP(k+1|k)H'+\sigma_v^2]^{-1} && \text{(ii)}\\
&\hat{\underline{x}}(k+1|k+1)=\phi\hat{\underline{x}}(k|k) + \underline{K}(k+1)[z(k+1)-H\phi\hat{\underline{x}}(k|k)] && \text{(iii)}\\
&P(k+1|k+1)=[I-\underline{K}(k+1)H]P(k+1|k) && \text{(iv)}
\end{aligned}\right\} \quad (8)$$

In order to use Eqs.(8), initial estimates for $\hat{\underline{x}}(k|k)$ and P(k|k) for some value of k are required.

If the position of the target is known exactly at three successive times t_k, k = 1, 2, 3, and the target was accelerating uniformly, the velocity $x_2(3)$ could be found from the relation

$$x_2(k) = \frac{1}{2T}[3x_1(3) - 4x_1(2) + x_1(1)] \qquad (9)$$

which may be derived from elementary kinematics. If instead the measured values of the range in Eq. (9) are used an estimate of the velocity at k = 3 is given by

$$\hat{x}_2(3|3) = \frac{1}{2T}[3z(3)-4z(2) + z(1)]$$

Similarly, the acceleration $x_3(3)$ would be given by (again from

elementary kinematics)

$$\hat{\underline{x}}_3(3|3) = \frac{1}{T^2}\,[z(3) - 2z(2) + z(1)]$$

The first value of k for which these estimates may be formed is k = 3; therefore, the earliest estimate of the state vector which can be made using the above procedure is

$$\hat{\underline{x}}_3(3|3) = \begin{bmatrix} z(3) \\ \frac{1}{2T}\,[3z(3)-4z(2)+z(1)] \\ \frac{1}{T^2}\,[z(3)-2z(2)+z(1)] \end{bmatrix} \qquad (10)$$

Because of the extreme sensitivity of $\hat{x}_3(3|3)$ to noise, a better (and simpler) estimate is given by

$$\hat{\underline{x}}(3|3) = \begin{bmatrix} z(3) \\ \frac{1}{2T}\,[3z(3)-4z(2)+z(1)] \\ 0 \end{bmatrix} \qquad (11)$$

To form an initial estimate of P(3|3), we will use Eq. (10). The state vector at k = 3 is represented by

$$\underline{x}(3) = \begin{bmatrix} x_1(3) \\ \frac{1}{2T}\,[3x_1(3)-4x_1(2)+x_1(1)] \\ \frac{1}{T^2}\,[x_1(3)-2x_1(2)+x_1(1)] \end{bmatrix} \qquad (12)$$

Combining Eqs. (6), (10), and (12),

$$\underline{x}(3)-\hat{\underline{x}}(3|3) = \begin{bmatrix} x_1(3) \\ \frac{1}{2T}\,[3x_1(3)-4x_1(2)+x_1(1)] \\ \frac{1}{T^2}\,[x_1(3)-2x_1(2)+x_1(1)] \end{bmatrix} - \begin{bmatrix} z(3) \\ \frac{1}{2T}\,[3z(3)-4z(2)+z(1)] \\ \frac{1}{T^2}\,[z(3)-2z(2)+z(1)] \end{bmatrix}$$

$$= -\begin{bmatrix} v(3) \\ \frac{1}{2T}\,[3v(3)-4v(2)+v(1)] \\ \frac{1}{T^2}\,[v(3)-2v(2)+v(1)] \end{bmatrix} \equiv -\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

From its definition and from Eqs. (4) and (7), $P(3|3)$ is given by

$$P(3|3)=E\left\{\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} [p_1 p_2 p_3]\right\} = E\left\{\begin{bmatrix} p_1^2 & p_1 p_2 & p_1 p_3 \\ p_1 p_2 & p_2^2 & p_2 p_3 \\ p_1 p_3 & p_2 p_3 & p_3^2 \end{bmatrix}\right\}$$

$$= \frac{\sigma_v^2}{2T^4}\begin{bmatrix} 2T^4 & 3T^3 & 2T^2 \\ 3T^3 & 13T^2 & 12T \\ 2T^2 & 12T & 12 \end{bmatrix} \qquad (13)$$

With Eqs. (11) and (13) as initial estimates of the state vector and covariance matrix, the state vector estimate is updated with each new measurement using the Kalman Eqs. (8). [1].

Filter B: Constant acceleration model. Synthetic time-varying gain.

The Kalman gain, $\underline{K}(k)$, computed from the second equation of the set (8) of the previous section, is a function of the measurement interval T, the measurement noise variance σ_v^2, the maneuver noise variance σ_w^2, and the iteration number k. To apply the Kalman equations to a particular filtering problem, these parameters must be known, and the Kalman gains may be calculated for each k independent of any measurement. The components of $\underline{K}(k)$ are therefore determined a priori.

A measurement interval of T = 0.04 seconds has been used throughout all simulations. For Filter A values of $\underline{K}(k)$ from

k = 3 to k = 200 were computed. It was supposed that each component of the Kalman gain could be approximated by a function of the form

$$K_i'(k) = \frac{a_i}{k^i - b_i} + K_i(\infty) \quad , \; i=1,2,3 \tag{14}$$

where the parameters a_i and b_i are found from fitting Eq. (14) to the values $K_i(5)$ and $K_i(30)$ which were computed for Filter A. $K_i(\infty)$ is the value to which $K_i(k)$ is an asymptote. Although Eqs. (14) do not purport to be the best possible fit to the optimum gain components, the advantage is that the $K_i(k)$ are easy to calculate.

If Eq. (14) is used to calculate the Kalman gain components, the state estimate is updated according to the single equation

$$\hat{\underline{x}}(k|k)=\phi\hat{\underline{x}}(k-1|k-1)+\underline{K}'(k)[z(k)-H\phi\hat{\underline{x}}(k-1|k-1)]$$

with a considerable saving in computation time.

<u>Filter C</u>: Constant Acceleration Model. Constant gain.

Figures (1), (2), and (3) show that each component of the Kalman gain assumes its largest value at k = 4, and decreases monotonically, asymptotically approaching some minimum value. The motivation of this filter is to find some particular value of k, say k = i, such that the constant gain $\underline{K}(i)$ may be used to update the state estimate for each iteration; i.e.,

$$\hat{\underline{x}}(k+1|k+1)=\phi\hat{\underline{x}}(k|k)+\underline{K}(i)[z(k+1)-H\phi\hat{\underline{x}}(k|k)]$$

If i is chosen to be small, the Kalman gain will be relatively large and will remain so, since $\underline{K}(i)$ is constant, by hypothesis. The resulting estimates will be very noisy and

show little tendency to converge. If the value of i chosen is large, the corresponding Kalman gain will be small and errors in the initial estmiates will be slowly corrected. It is hoped that an intermediate value of i can be found such that after a reasonable number of iterations, convergence can be achieved.

Filter D: Constant Velocity Model. Time varying gain.

As its name suggests, this filter assumes that the target is moving at constant velocity. The development of the filter equations for this model is parallel to that of Filter A. It is simpler in the sense that the state vector has two components rather than three. The associated matrices are hence 2x2, rather than 3x3.

Let the state vector of the target be denoted by

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ represents the range of the target and $x_2(t)$ is its (radial) velocity. This model assumes that the state vector evolves according to

$$x_1 = x_2$$
$$\dot{x}_2 = w(t)$$

where w(t) is assumed to be zero-mean, Gaussian noise uncorrelated with itself in time. In matrix notation,

$$\dot{\underline{x}}(t) = F\underline{x}(t) + Gw(t) \tag{15}$$

where

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad , \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The solution of Eq. (15) is given by

$$\underline{x}(t)=\phi(t,t_o)\underline{x}(t_o) + [\int_{t_o}^{t} \phi(t,\tau)G(\tau)d\tau]w(t) \tag{16}$$

where

$$\phi(t,\tau) = \begin{bmatrix} 1 & t-\tau) \\ 0 & 1 \end{bmatrix}$$

For successive measurement times t_{k+1} and t_k, Eq. (16) becomes

$$\underline{x}(t_{k+1})=\phi(t_{k+1}, t_k) +[\int_{t_k}^{t_{k+1}} \phi(t_{K+1},\tau)G(\tau)d\tau] \; w(t_k) \tag{17}$$

Defining, as before

$$\underline{x}(t_{k+1}) \equiv \underline{x}(k+1)$$

$$\underline{x}(t_k) \equiv \underline{x}(k)$$

$$\int_{t_k}^{t_{k+1}} \phi(t_{k+1}, \tau)G(\tau)d\tau \equiv \Gamma(k+1, k)$$

With the above definitions, Eq. (16) may be written as

$$\underline{x}(k+1)=\phi(k+1, k)\underline{x}(k) + \Gamma(k+1, k)w(k)$$

and by direct integration,

$$\Gamma(k+1, k)=\int_{t_k}^{t_{k+1}} \phi(t_{k+1}, \tau)G(\tau)d\tau= \begin{bmatrix} \frac{1}{2} T^2 \\ T \end{bmatrix}$$

where

$$T \equiv t_{k+1} - t_k$$

and

$$\phi(k+1,\ k) \equiv \phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

which is constant, given a fixed interval between successive measurements.

Again, assume that

$$E\left\{ w(j)w(k) \right\} = Q(k)\delta_{jk}$$

where

$$Q(k) = \sigma_w^2 \qquad \text{(maneuver noise variance)}$$

The measurement model is, as before

$$z(k+1) = H\underline{x}(k+1)+v(k+1) \tag{18}$$

where $v(k+1)$ is the (scalar) measurement noise and

$$H = [\ 1 \quad 0\ ]$$

so that eq. (18) may be written

$$z(k+1) = x_1(k+1)+v(k+1)$$

Again, assume that the measurement noise has the properties

$$E\left\{ v(j)v(k) \right\} = R(k)\delta_{jk} \qquad R(k)=\sigma_v^2$$

and

$$E\left\{ v(j)w(k) \right\} = 0$$

The Kalman equations for this model have the same form as Eq. (8), where all quantities in those equations assume the definitions given in this section. The initial state estimate and covariance matrix are initialized for k = 2 as follows:

$$\hat{\underline{x}}(2|2) = \begin{bmatrix} z(2) \\ \frac{1}{T}\,[z(2)-z(1)] \end{bmatrix} \tag{19}$$

and

$$P(2|2) = \frac{\sigma_v^2}{T^2} \begin{bmatrix} T^2 & T \\ T & 2 \end{bmatrix} \tag{20}$$

$P(2|2)$ is obtained in a manner analogous to the procedure used for Filter A.

With these initial estimates of the state vector and covariance matrix, the state vector estimate is updated with each new measurement by means of the Kalman Eqs. (8). [1].

3.1.3 Performance

Two types of trajectories were used in the simulations: those resulting from a constant acceleration (called constant acceleration trajectories) and those resulting from a sinusoidal acceleration (called sinusoidal acceleration trajectories). A constant acceleration trajectory conforms to the model of Filters A, B, and C, but is a departure from the model of Filter D, except for $x_3(k) = 0$. A sinusoidal acceleration trajectory is a departure from the models of all filters. This type of trajectory serves to determine the value of σ_w^2 (maneuver noise variance) which will optimize the response of the filter in the sense that it will track departures from the model, yet not be too sensitive to measurement noise.

A trajectory is completely determined by the form of the acceleration, $x_3(k)$, the initial range $x_1(0)$, and the initial velocity $x_2(0)$. Range, velocity, and acceleration are then

computed for each k. After generating the true values $x_1(k)$, $x_2(k)$, and $x_3(k)$, a noise term is computed as follows.

Twelve random numbers, each between 0 and 1, are generated and added. The number 6 is then subtracted from the sum. The result, when multiplied by σ_v, is an approximately zero-mean, Guassian random variable with standard deviation σ_v. In symbols,

$$v(k)=\sigma_v \left\{ \sum_{i=1}^{12} N_i - 6 \right\} = \sigma_v n_k \quad 0 \leq N_i < 1$$

where N_i is a random number between 0 and 1.

Starting with k = 1, the same sequence of random variables n_k was generated for all runs. All variable acceleration simulations used the same sequence of random variables; i.e., for a given k, the value of n_k was always the same. The constant acceleration trajectories use the first 50 values of n_k of this set to compute the noise term, v(k).

Figure 1 is a plot of v(k) for k = 1 to 50 for a standard deviation σ_v=6 m. Table I lists the values of v(k) from k = 1 to 50 for σ_v=6 m. Note the preponderance of negative noise terms in the last few entries. The average of the last eight noise terms is -2.59 m and the average of the last six is -2.99 m. This results in a negative bias in the estimates for the last several terms of the constant acceleration simulations.

Measurement noise characterized by $\sigma_v=\{2, 4, 6, 8, 10 \text{ m}\}$ was used in these simulations. A study by Oricon [2] suggests a value of σ_v=3 m for radar of "medium accuracy", but σ_v=6 m was chosen as a representative of the measurement noise encountered at range of about 600 m. Section 3.1.3 of this report shows σ_v

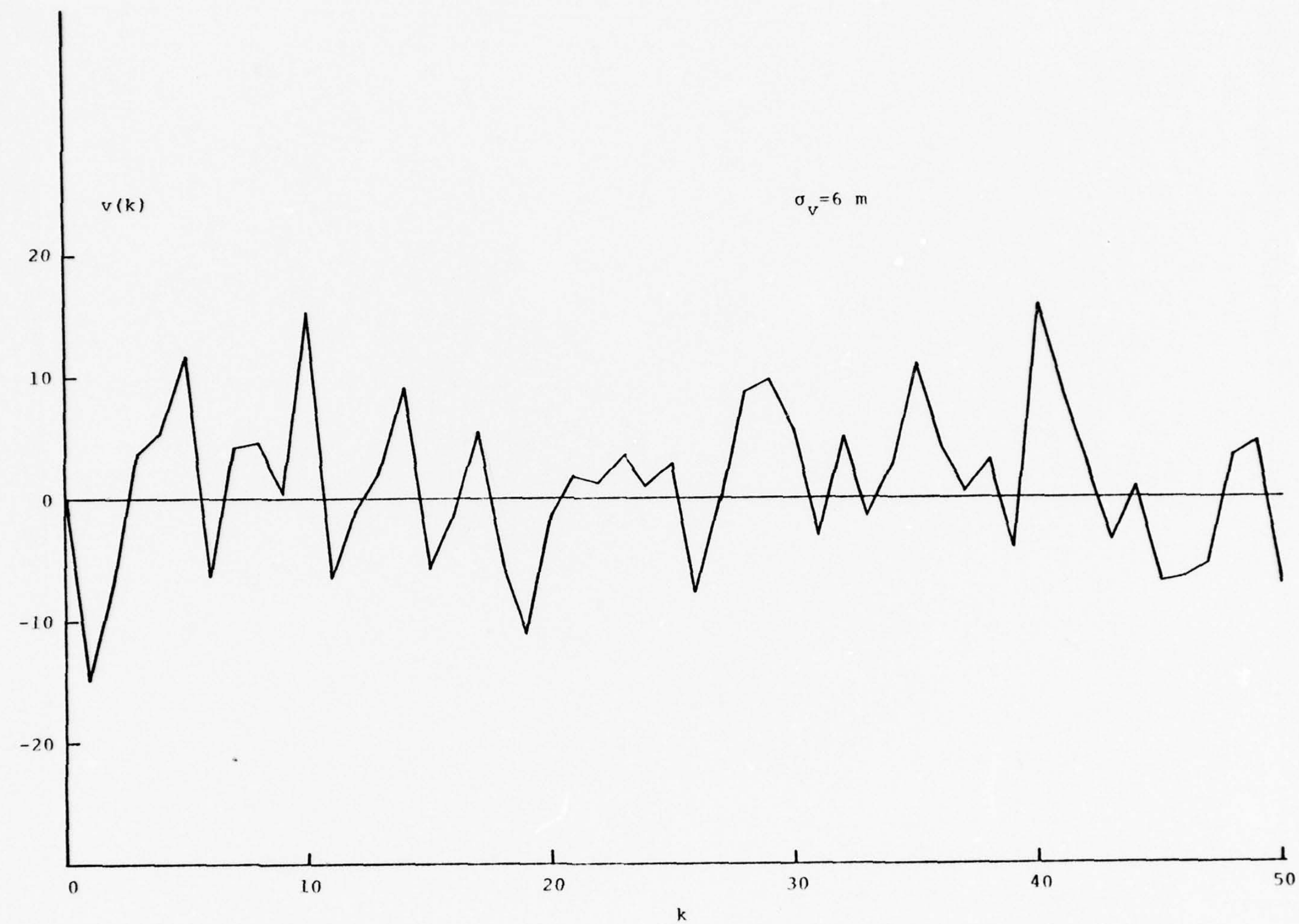


Figure 1. Simulated Measurement Noise

TABLE 1

NOISE TERMS v(k)

$\sigma_v = 6m$

k	v(k)	
1	-14.75	
2	- 6.92	
3	3.74	
4	5.56	
5	11.82	
6	- 6.24	
7	4.24	
8	4.65	
9	.40	
10	15.38	
11	- 6.57	
12	- .99	
13	2.20	
14	8.97	
15	- 5.79	
16	- 1.33	
17	5.55	
18	- 5.28	
19	-10.98	
20	- 1.42	
21	1.87	
22	1.12	
23	3.37	
24	1.08	
25	2.88	
26	- 7.73	
27	- .15	
28	8.57	
29	9.49	
30	5.68	
31	- 3.06	
32	5.00	
33	- 1.37	
34	2.81	
35	11.02	
36	4.19	
37	.53	
38	3.00	
39	- 4.03	
40	15.76	
41	8.69	
42	2.81	
43	- 3.33	
44	.70	
45	- 6.93	region of
46	- 6.49	negative bias
47	- 5.18	
48	3.20	
49	4.66	
50	- 7.19	

to be about 6 m for the MA-1 radar and to be only weakly range dependent.

Table 2 is a summary of simulated trajectories and noise variances used to evaluate the filters. The line numbers at the left identify the trajectory and the filter parameters used. At the right of the table, the letter A, B, C, or D indicates the filters used to estimate state variables from the specified trajectory. A hyphen (-) indicates the filter was not used, for reasons which will be presented later.

Plots of filter performance are to be found in Appendix A. Each figure in this appendix is a plot of the true velocity and acceleration and their estimates by each of the four filters. The true velocity and acceleration are plotted with solid lines, and their corresponding estimates are plotted with dashed lines. Since Filter D does not yield an acceleration estimate, no such plot will, of course, be present. Since Filter C was used only in cases 16, 17, 18, 24, and 30, estimates from this filter will be present in these cases and absent in all others.

The format of each figure is as follows. The numerical part of the figure number is the same as the case number in Table 2. The plots in the left column of each figure show $x_2(k)$ and $\hat{x}_2(k|k)$ vs k for each filter used. There will be either three or four such plots in this column, depending on whether or not Filter C was used for that case. The right column of each figure shows plots of $x_3(k)$ and $\hat{x}_3(k|k)$ vs k for Filters A and B and for Filter C as well in those cases where this filter was used. There will be either two or three such plots in the right column.

TABLE 2

Simulated Trajectories and Noise Variances

Case	Range	Velocity	Acceleration	Measurement Noise Variance	Maneuver Noise Variance	Filter
	$x_1(0)$ (m)	$x_2(0)$ $m\,s^{-1}$	$x_3(k)$ $m\,s^{-2}$	σ_v^2	σ_w^2	
1	600	200	0	4	0	A B - D
2	"	"	9.8	"	"	A B - D
3	"	"	58.8	"	"	A B - D
4	"	"	0	16	"	A B - D
5	"	"	9.8	"	"	A B - D
6	"	"	58.8	"	"	A B - D
7	"	"	0	36	"	A B - D
8	"	"	9.8	"	"	A B - D
9	"	"	58.8	"	"	A B - D
10	"	"	0	64	"	A B - D
11	"	"	9.8	"	"	A B - D
12	"	"	58.8	"	"	A B - D
13	"	"	0	100	"	A B - D
14	"	"	9.8	"	"	A B - D
15	"	"	58.8	"	"	A B - D
16	"	"	0	36	6000	A B C D
17	"	"	9.8	"	"	A B C D
18	"	"	58.8	"	"	A B C D
19	"	"	-9.8	"	0	A B - D
20	"		-58.8	"	"	A B - D
21	"	"	$2g \sin\frac{\pi k}{100}$	36	0	A B - D
22	"	"	"	"	2000	A B - D
23	"	"	"	"	4000	A B - D
24	"	"	"	"	6000	A B C D
25	"	"	"	"	8000	A B - D
26	"	"	"	"	10000	A B - D
27	"	"	$6g \sin\frac{\pi k}{100}$	"	0	A B - D
28	"	"	"	"	2000	A B - D
29	"	"	"	"	4000	A B - D
30	"	"	"	"	6000	A B C D
31	"	"	"	"	8000	A B - D
32	"	"	"	"	10000	A B - D

Thus, Figure A-15 contains plots of the performance of Filters A, B, and C for the trajectory (and noise variances) listed under Case 15; i.e., $x_1(0) = 600$ m, $x_2(0) = 200$ m, $x_3(k) = 58.8$ m s^{-2}, $\sigma_v^2 = 100$ m^2, $\sigma_w^2 = 0$.

Cases 1 through 20 of Table 2 list constant acceleration simulations. These conform to the model of Filters A, B, and C. Cases 1, 4, 7, 10, 13, and 16 also conform to the model of Filter D.

In Cases 1 through 15, $\sigma_w^2 = 0$ and measurement noise variances are given the values $\sigma_v^2 = \{4, 16, 36, 64, 100 \text{ m}^2\}$. In Cases 16 through 18, $\sigma_v^2 = 36$ m^2 and $\sigma_w^2 = 6000$ $m^2 s^{-6}$; the values judged "best" to handle deviations from the model. Data supporting this conclusion are listed in Section 3.1.4. Cases 19 through 20 specify negative accelerations. Cases 21 through 26 specify sinusoidal accelerations of 2g amplitude and 8 s period. Cases 27 through 32 specify sinusoidal accelerations of the same period, but with a 6g amplitude.

Filter A: Constant Acceleration Model. Time varying gain.

Figure A-1 through A-20 contain plots of the velocity and acceleration estimates of Filter A for Cases 1 through 20 listed in Table 2. The value of σ_v^2 was used in both the noise generator and in the filter. These plots show both velocity and acceleration estimates to be extremely noisy for the first half-second, but they smooth out considerably at the end of one second, and approach the true values quite closely after about 30 iterations (1.2 s). As expected, the lower the value of σ_v^2, the faster and closer the convergence. Note the characteristic "tail", or

negative bias, in the estimates, particularly those of higher signal noise variance. This is caused by the anomoly in the random numbers referred to earlier.

Figure A-21 through A-32 contain plots of velocity and acceleration estimates of sinusoidal acceleration simulations. Several values of the maneuver noise variance are used to find the "best" value of σ_w^2. (See Section 3.1.4.) It is clear from Figure A-21 and A-27 that filter estimates using $\sigma_w^2 = 0$ are extremely poor, as one would expect.

<u>Filter B</u>: Constant Acceleration Model. Synthetic gain.

The motivation of this filter and the method of calculation of the parameters used to generate $\underline{K}'(k)$ are discussed in Section 3.1.2. Since the values of $\underline{K}(k)$, the optimum time-varying gain, depend upon the noise parameters σ_v^2 and σ_w^2, the parameters which generate $\underline{K}'(k)$ were found by fitting Eq. (14) to $K_i(5)$ and $K_i(30)$ for each pair of noise parameters.

Figures 2, 3, and 4 are plots of $K_i(k)$ and $K_i'(k)$ vs k for the three components of the Kalman gain. The noise parameters used to generate $\underline{K}(k)$ were those judged to be optimum in a sense discussed in the next section; $\sigma_v^2 = 36\ m^2$ and $\sigma_w^2 = 6000\ m^2s^{-6}$. Although the fit is not a close one, this procedure yields synthetic Kalman gains such that the estimates of Filter B in Figures A-1 through A-32 are difficult to distinguish from corresponding plots of Filter A.

<u>Filter C</u>: Constant Acceleration Model. Constant gain.

Figures 2, 3, and 4 are plots of the components of the optimum Kalman gain vs the iteration number k. The optimum constant

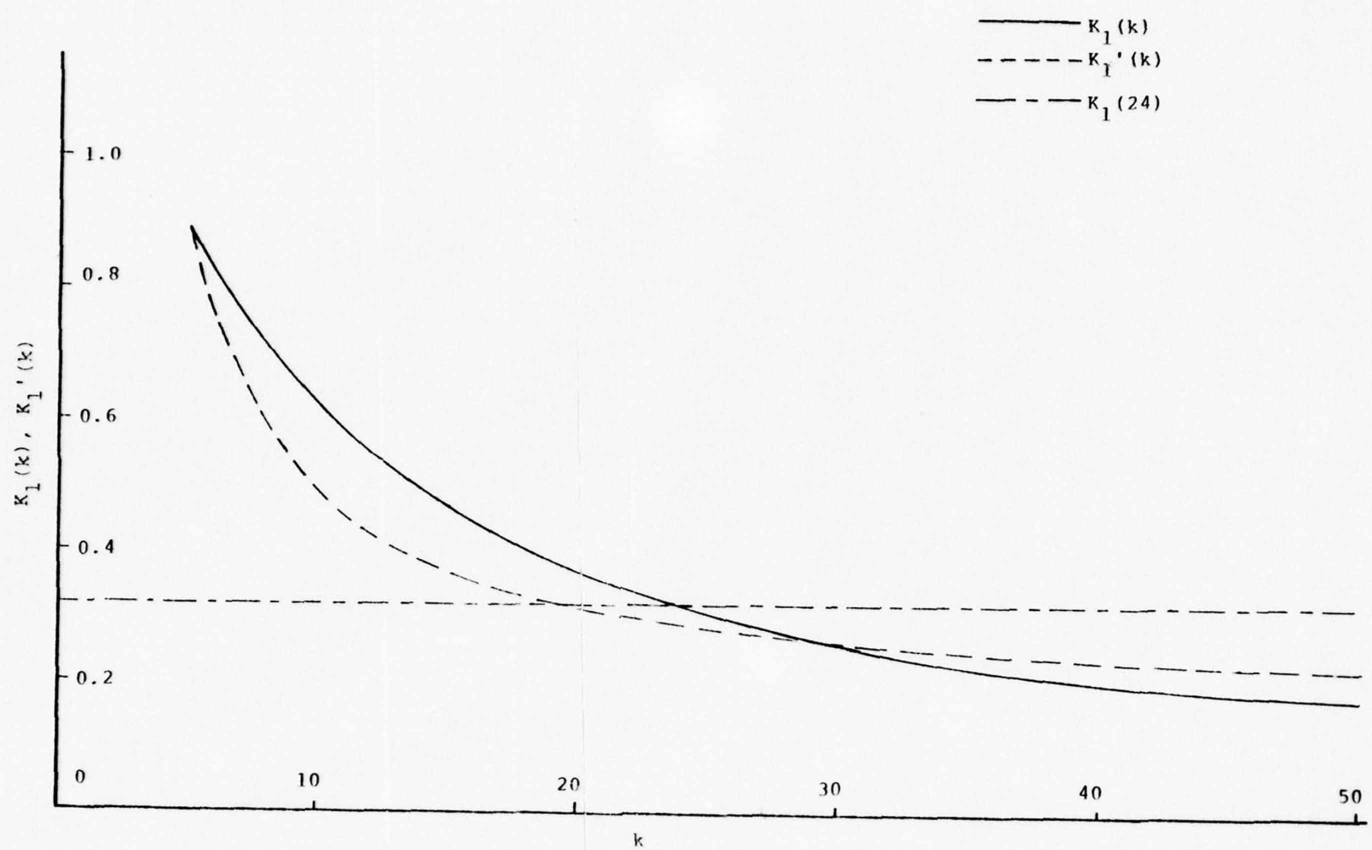


Figure 2. Position Kalman Gain Component

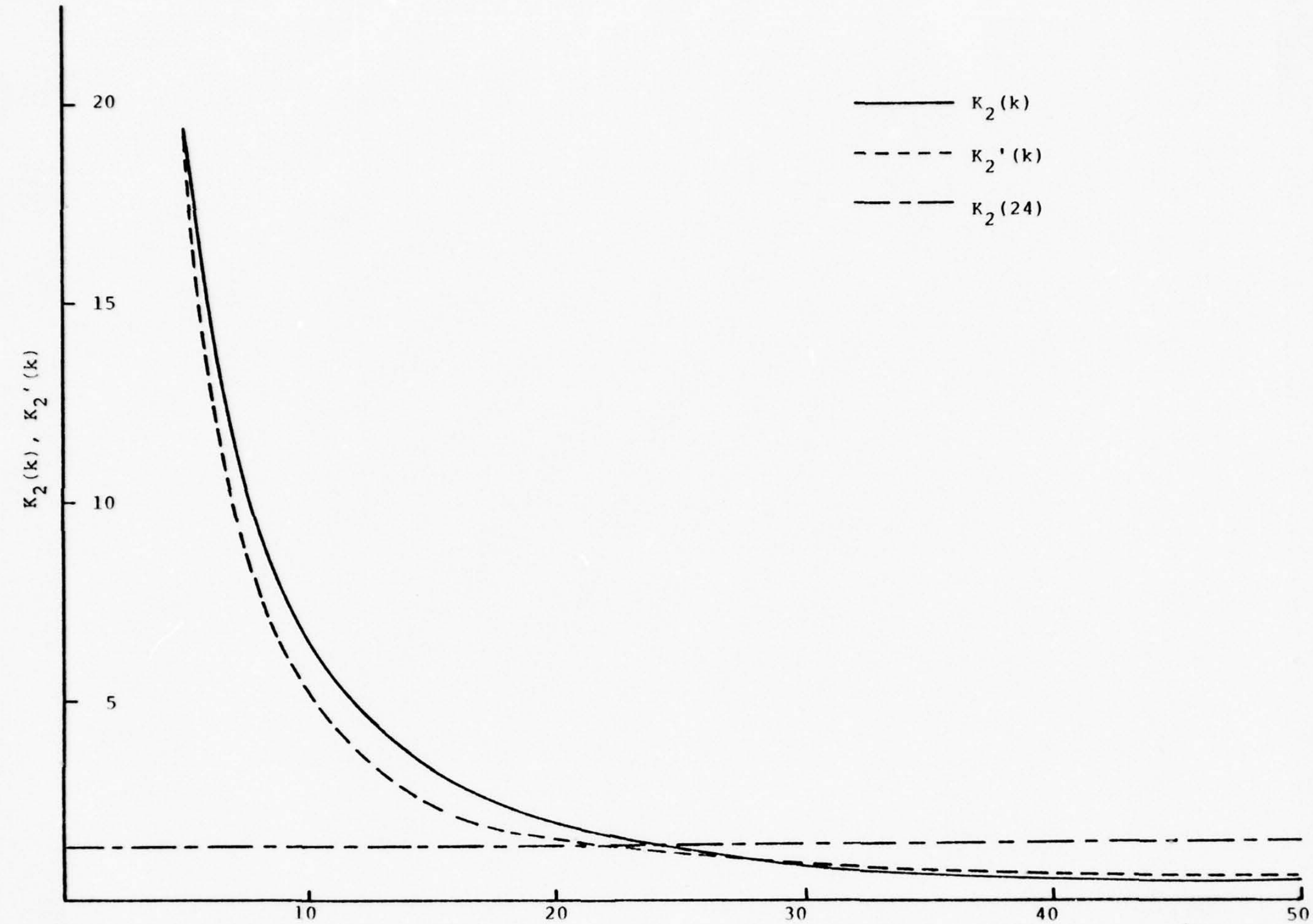


Figure 3. Velocity Kalman Gain Component

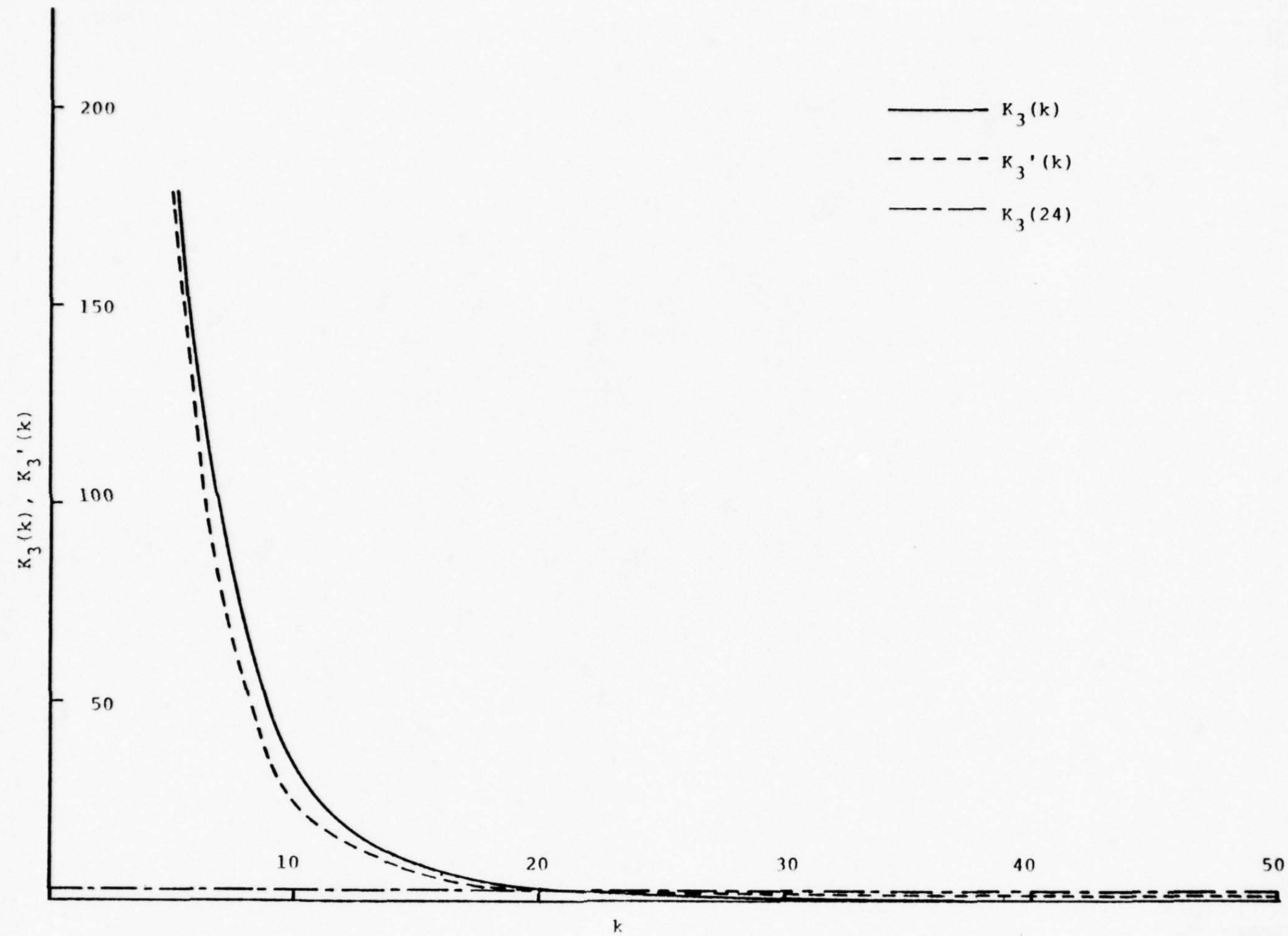


Figure 4. Acceleration Kalman Gain Component

value of the Kalman gain $\underline{K}(i)$ was determined by finding the gain which minimizes the rms deviation of the velocity. (See Section 3.1.4.) This gain was found to be $\underline{K}(24)$, and is shown in Figures 2, 3, and 4 as a dotted horizontal line. Since these values were based on optimum gains for $\sigma_v^2 = 36\ m^2$ and $\sigma_w^2 = 6000\ m^2 s^{-6}$, we used Filter C only where appropriate; i.e., for parameters listed on lines 16, 17, 18, 24, and 30. Figure A-16, A-17, and A-18 show that velocity estimates of Filter C tend to converge to the true values, though not as quickly nor as closely as the corresponding velocity estimates of Filters A and B. Figure A-16, A-17, and A-18 show that the acceleration estimates of Filter C also show a tendency to converge, but more weakly than those of Filters A and B. Figure A-24 and A-30 show noisier velocity estimates than those shown in Figures A and B, and the acceleration estimates shown in these figures are much noisier.

<u>Filter D</u>: Constant Velocity Model.

This filter assumes the target to be moving with constant relative velocity. Departures from constant velocity can be treated by choosing a suitable value of σ_w^2. This filter yields position estimates and velocity estimates only. Attempts to estimate acceleration based on some algorithm using past velocity estimates proved fruitless.

It can be seen from the figures that even for very large values of σ_w^2, this filter yields a biased velocity estimate even for accelerations of only 1g, which becomes considerably larger for 6g accelerations. The velocity estimates are quite poor for sinusoidal acceleration simulations, though better than Filter C.

3.1.4 Evaluation

Given a trajectory and a pair of noise variances (σ_v^2 and σ_w^2) we wish to characterize the performance of each filter by a single number. A root-mean-square deviation is an attractive candidate for such a characteristic number. The first difficulty with this idea, however, is that the filter output consists of three time-varying quantities $\hat{x}_1(k|k)$, $\hat{x}_2(k|k)$, and $\hat{x}_3(k|k)$; i.e., a range estimate, a velocity estimate, and an acceleration estimate. The smallest rms deviation for one component of the state estimate may not be the smallest for the others. Another difficulty can be seen by comparing plots of the estimates of Filter A with those of Filter D. Although Filter A produces noisy velocity and acceleration estimates for low k, the estimates are quite good for large k (constant acceleration trajectories), with the exception of the "tail" referred to earlier. Although Filter D yields a less noisy velocity estimate for low k, it is biased and slowly diverges from the true velocity for large k. Thus the choice of the better filter depends upon the allowable time between acquisition and when estimates are needed. For the constant acceleration trajectories, we have assumed this interval to be 1.5 sec. Thus, if the velocity rms deviation were computed over the 2 sec. interval, one would find the deviation lower for Filter D than for Filter A, due to the high noise in Filter A for small k. Such a procedure would lead to Filter D being judged "best". But if one has at least 1.5 sec. between acquisition and the time an estimate is needed, a comparison of the estimates of Filter A with those of Filter D shows that Filter A is best. Since $\hat{x}_2(k|k)$ and $\hat{x}_3(k|k)$ of Filter A smooth out in about 1.5 s.

it was decided to compute the rms estimates from k = 39 (1.56 s) to k = 50 (2.00 s). In symbols,

$$(\Delta x_i)_{rms} = \sqrt{\sum_{k=39}^{50} [x_i(k) - x_i(k|k)]^2} \qquad i=1,2,3$$

where the subscript i refers to one of the three components of the state vector and the state estimate. Results are listed in Tables 3, 4, and 5.

For sinusoidal acceleration simulations, which were 8 sec. in duration or from k = 1 to k = 200, the figures show that Filters A, B, and D give reasonable estimates after about 3 sec., so the rms deviations were computed from k = 75 (3 sec.) to k = 200 (8 s); i.e.,

$$(\Delta x_i)_{rms} = \sqrt{\sum_{k-76}^{200} [x_i(k) - x_i(k|k)]^2} \qquad i=1,2,3.$$

Range Estimates. $(\Delta x_1)_{rms}$

This, of course, is the most important of the three estimates, but a reasonable estimate is not difficult to obtain.

Cases 1 through 20 of Table 3 list the rms range deviations for constant acceleration trajectories. It can be seen that Filter A characteristically has the smallest deviations, with Filter B a close second. The deviations of Filter C are about 25% higher than Filter A. The deviations of Filter D are quite erratic; small for small values of the acceleration and much larger for large acceleration. This is to be expected, for Filter D is based on a constant velocity model.

TABLE 3 RMS Range Deviations

Case #	$x_1(0)$	$x_2(0)$	$x_3(k)$	σ_v^2	σ_w^2	$(\Delta x_1)_{rms}$ Filter A	B	C	D
1	600	200	0	4	0	1.01	1.09	1.27	3.18
2			9.8			"	"	"	1.55
3			58.8			1.00	1.08	"	12.07
4			0	16		2.02	2.17	2.54	3.18
5			9.8			"	"	"	1.55
6			58.8			2.01	"	"	12.07
7			0	36		3.03	3.26	3.80	3.18
8			9.8			"	"	5.43	1.55
9			58.8			"	"	"	12.07
10			0	64		4.04	4.35	5.07	3.18
11			9.8			"	"	"	1.55
12			58.8			4.03	4.34	"	12.07
13			0	100		5.05	5.44	6.34	3.18
14			9.8	"		"	"	"	1.55
15			58.8	"		5.04	5.43	"	12.08
16			0	36	6000	3.13	4.01	3.80	3.37
17			9.8			"	"	"	2.96
18			58.8			3.12	----	3.81	3.37
19			-9.8		0	3.03	3.26	----	5.47
20			-58.8			3.04	----	----	17.63
21		0	$2g\sin\frac{\pi k}{100}$	"	0	16.04	16.39	----	16.74
22			"	"	2000	1.94	1.46	----	1.92
23			"	"	4000	1.93	2.01	----	1.87
24			"	"	6000	1.97	2.06	2.98	1.95
25			"	"	8000	2.01	2.10	----	2.03
26			"	"	10000	2.05	2.13	----	2.10
27			$6g\sin\frac{\pi k}{100}$		0	50.67	52.33		50.72
28			"		2000	3.81	3.43	----	4.78
29			"		4000	3.00	2.87	----	3.40
30			"		6000	2.70	2.66	2.98	2.91
31			"		8000	2.56	2.56	----	2.69
32			"		10000	2.49	2.50	----	2.58

TABLE 4 RMS Velocity Deviations

Case #	$x_1(0)$	$x_2(0)$	$x_3(k)$	σ_v^2	σ_w^2	$(\Delta x_2)_{rms}$ Filter A	B	C	D
1	600	200	0	4	0	2.13	2.07	5.43	2.20
2			9.8			"	"	5.42	6.86
3			58.8			2.15	2.08	5.38	49.53
4			0	16		4.26	4.15	10.9	2.20
5			9.8			"	"	"	6.86
6			58.8			4.28	4.16	10.8	49.53
7			0	36		6.39	6.21	16.3	2.20
8			9.8			"	6.22	"	6.86
9			58.8			6.40	6.23	16.2	49.53
10			0	64		8.52	8.29	21.7	2.20
11			9.8			"	"	"	6.86
12			58.8			8.54	8.30	"	49.53
13			0	100		10.64	10.36	27.2	2.20
14			9.8			10.65	"	"	6.86
15			58.8			10.67	10.37	27.1	49.53
16			0	36	6000	7.21	9.55	16.3	6.58
17			9.8			"	"	"	8.29
18			58.8			7.22	"	16.2	2.50
19			-9.8		0	6.38	----	----	10.44
20			-58.8			6.37	----	----	53.14
21		0	$2g\sin\frac{\pi k}{100}$		0	28.24	29.74	----	22.23
22			"		2000	6.03	6.36	----	7.31
23			"		4000	5.49	5.79	----	6.71
24			"		6000	5.44	5.67	12.59	6.69
25			"		8000	5.42	5.72	----	6.85
26			"		10000	5.65	5.82	----	7.06
27			$6g\sin\frac{\pi k}{100}$		0	86.54	92.08	----	66.80
28			"		2000	16.17	17.39	----	20.50
29			"		4000	13.13	14.13	----	17.14
30			"		6000	11.71	12.52	12.99	15.53
31			"		8000	10.88	11.61	----	14.67
32			"		10000	10.34	10.95	----	14.07

TABLE 5 RMS Acceleration Deviations

Case #	$x_1(0)$	$x_2(0)$	$x_3(k)$	σ_v^2	σ_w^2	$(\Delta x_3)_{rms}$ Filter A	B	C	D
1	600	200	0	4	0	2.52	2.52	9.13	
2			9.8			2.53	2.53	9.11	
3			58.8			2.58	2.58	9.06	
4			0	16		5.04	5.03	18.26	
5			9.8			5.05	5.04	18.25	
6			58.8			5.10	5.09	18.18	
7			0	36	0	7.56	7.55	27.4	
8			9.8			7.57	7.56	"	
9			58.8			7.62	7.61	27.3	
10			0	64		10.08	10.06	36.52	
11			9.8			10.09	10.07	36.51	
12			58.8			10.14	10.12	36.44	
13			0	100		12.60	12.58	45.65	
14			9.8			12.61	12.59	45.64	
15			58.8			12.66	12.64	45.57	
16			0	36	6000	9.21	11.14	27.39	
17			9.8			9.22	"	27.38	
18			58.8			9.25	"	27.31	
19			-9.8		0	7.55	----	----	
20			-58.8		0	7.50	----	----	
21		0	$2g\sin\frac{\pi k}{100}$		0	20.33	20.39	23.14	
22			"		2000	11.46	11.87	"	
23			"		4000	10.64	11.02	"	
24			"		6000	10.36	10.68	"	
25			"		8000	10.30	10.58	"	
26			"		10000	10.36	10.59	"	
27			$6g\sin\frac{\pi k}{100}$		0	61.94	62.63	28.56	
28			"		2000	33.13	34.93	"	
29			"		4000	29.83	31.24	"	
30			"		6000	28.03	29.26	"	
31			"		8000	26.86	27.99	"	
32			"		10000	26.02	27.03	"	

Lines 21 through 32 of Table 3 list $(\Delta x_1)_{rms}$ for the sinusoidal acceleration simulations. Trajectories of 6g amplitude represent a more extreme departure from the constant acceleration model than are likely to be encountered in practice, and are included to test filter response. Trajectories of 2g amplitude are more realistic, and therefore greater weight is given to these cases.

Filter B has values of $(\Delta x_1)_{rms}$ only a few per cent greater than Filter A, and the deviations of Filter D are even less than those of Filter B. Filter C contains the implicit parameters $\sigma_v^2 = 36\ m^2$ and $\sigma_w^2 = 6000\ m^2\ s^{-6}$, and was used only for trajectories listed on lines 24 and 30. The deviation is notably poorer than that of the other filters.

<u>Velocity Estimates</u>. $(\Delta x_2)_{rms}$

Velocity estimates are more sensitive to noise than range estimates, and $(\Delta x_2)_{rms}$ will vary more widely for different filters than $(\Delta x_1)_{rms}$. Lines 1 through 20 of Table 4 list $(\Delta x_2)_{rms}$ of the velocity estimates $\hat{x}_2(k|k)$. For nearly all cases, deviations of Filters A and B are more nearly the same. In fact, for most cases, the deviations of Filter B are slightly less than those of Filter A, but this result is most likely anomolous. Filter D yields erratic deviations; small for trajectories which match its model and much larger for those which do not. Filter C yields consistently large deviations.

Lines 21 through 32 list $(\Delta x_2)_{rms}$ for the sinusoidal acceleration simulations. As above, Filters A and B differ

but slightly and Filter C has much larger deviations, but the deviations of Filter D are surprisingly small, though not so small as those of Filter B.

Acceleration Estimates. $(\Delta x_3)_{rms}$

Lines 1 through 20 of Table 5 list $(\Delta x_3)_{rms}$ for Filter A, B, and C. There are no deviations computed for Filter D, for this filter does not provide an acceleration estimate. The deviations of Filters A and B are nearly the same, while those of Filter C are about three times greater.

Lines 21 through 32 list $(\Delta x_3)_{rms}$ for sinusoidal acceleration simulations. For the 2g amplitude trajectories, Filters A and B have deviations of about 1g and Filter C has deviations about twice as great.

Selection of Maneuver Noise Variance

As noted earlier, the maneuver noise variance (σ_w^2) is needed to allow the filter to respond to deviations from the model on which it is based. Of all simulations, only the sinusoidal trajectories (Cases 21 through 32) are deviations from the models of all filters, and so the best value of σ_w^2 is that which minimizes $(\Delta x_2)_{rms}$ and $(\Delta x_3)_{rms}$ for these cases. Since the 2g simulations are more typical of an encounter than the 6g simulations, the choice of a value of σ_w^2 will be based upon the rms deviations resulting from Cases 21 through 26 of Tables 4 and 5. For Filter A, the minimum rms deviations for both $(\Delta x_2)_{rms}$ and $(\Delta x_3)_{rms}$ occurs for $\sigma_w^2 = 8000\ m^2\ s^{-6}$.

For Filter B, the minimum $(\Delta x_2)_{rms}$ occurs for $\sigma_w^2 = 6000\ m^2\ s^{-6}$ and the minimum $(\Delta x_3)_{rms}$ occurs for $\sigma_w^2 = 8000\ m^2\ s^{-6}$. Since the differences in the rms deviations resulting from these two choices are quite small, $\sigma_w^2 = 6000\ m^2\ s^{-6}$ was chosen, for the use of the smaller value of the maneuver noise variance will result in a slightly less noisy filter.

3.1.5 Conclusion

The most striking feature of the tables of rms deviations is that in nearly all cases the deviations of Filters A and B are within a few percent of one another. Filter A computes the Kalman gain components from Eqs. (i) and (iv) of equation set (8) by matrix multiplications, whereas Filter B computes the gain components from three simple algebraic equations.

Filter A requires the parameter σ_v^2, σ_w^2, and T in order to give numerical results. If these parameters are given, Eqs. (8i), (8ii), and (8iv), together with Eq. (13) - initialization of the covariance matrix - can be used to calculate $\underline{K}(k)$, independently of any measurements. By following the procedure described in Section II, the parameters a_i, b_i, and $K_i(\infty)$ can be found for i = 1, 2, and 3. The Kalman gains can be quickly computed from Eqs. (21) and these nine parameters at a fraction of the time needed by the iterative procedure of Eqs. (8) to yield estimates of only slightly lower quality than those of the optimum filter, Filter A.

Although the calculations employed by Filter C are even simpler than those needed for Filter B, the results are much poorer and it is doubtful that the estimates would be useful.

Filter D cannot produce a useful acceleration estimate, and its performance in estimating velocities is very poor at accelerations higher than 1g.

[1] Stochastic Optimal Linear Estimation and Control, J.S. Meditch, McGraw-Hill(1969).

[2] Advanced Defensive Fire Control Processing Methods Study. ORINCON F33615-76-C-1303 Contract Status Briefing 31 Jan 77.

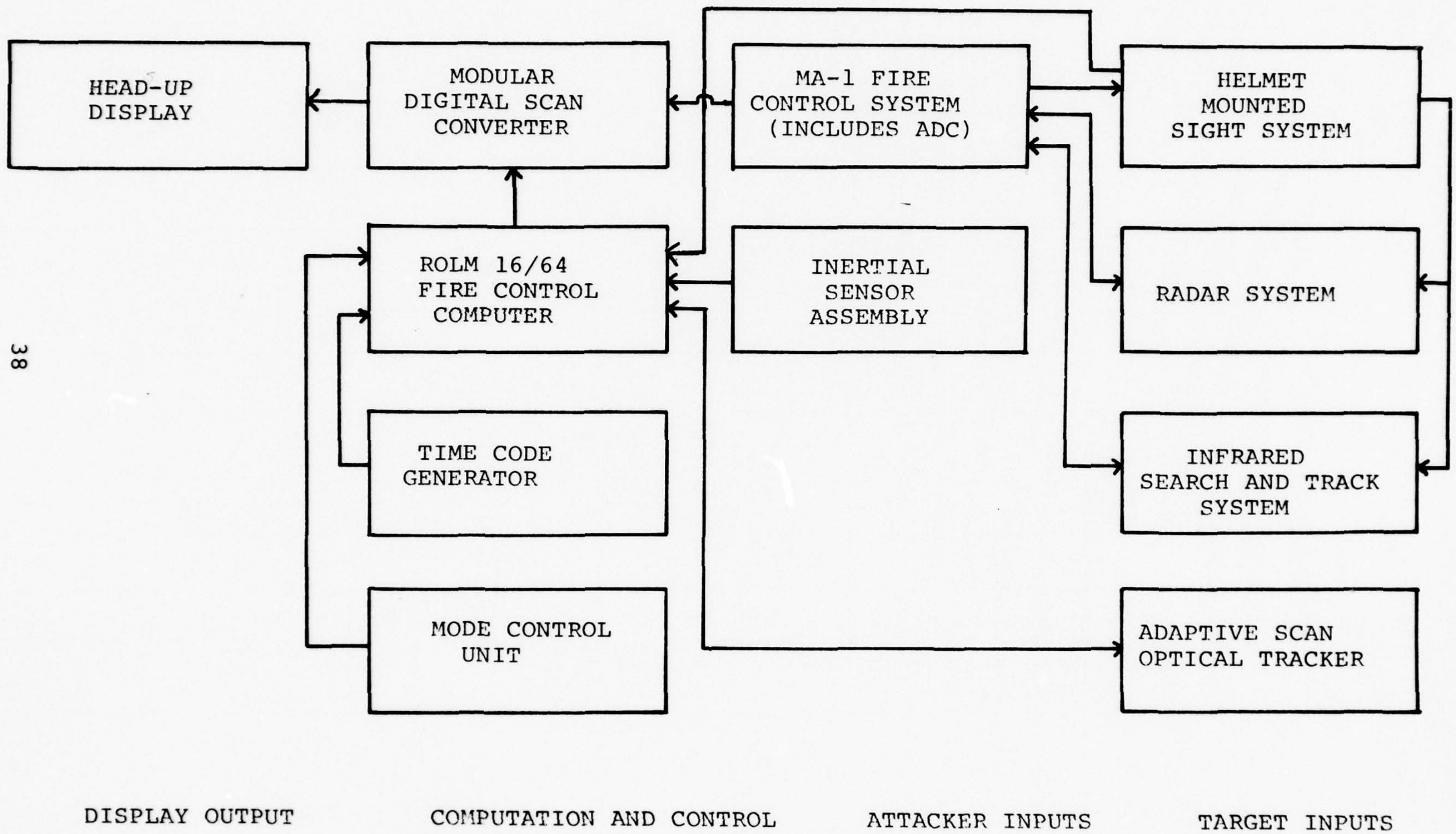
3.2 TARGET STATE MEASUREMENT

3.2.1 Introduction

The general objective of the target state measurement task is to devise independent methods for evaluating the accuracy of target state vectors (position, velocity, and acceleration) obtained from a director fire control system. The particular director system to be considered is being developed by Honeywell (the integrating contractor) for AFAL/RWT. A simplified block diagram of some essential features of this system is shown in Figure 5; components and subsystems which would not affect an actual combat encounter (e.g., the Mini-Heads-Up Display, used to obtain film records) are not included in this diagram. Director fire control systems are characterized by an extensive use of data in a line-of-sight coordinate frame rather than an aircraft body frame, and this characteristic may lead, in principle, to a more stable gunsight system. Figure 5 shows that the attacker's Inertial Sensor Assembly and Air Data Computer (via the MA-1 Fire Control System) are inputs to the director system in addition to the target sensors and the Time Code Generator and Mode Control Unit. The final output of the director system is presented on the Heads-Up Display.

The task objective stated above has several possible interpretation features. First, the stage in the director system at which target position, velocity, and acceleration are to be measured is not specified. Presumably, in order to evaluate

FIGURE 5. SIMPLIFIED BLOCK DIAGRAM OF THE DIRECTOR FIRE CONTROL SYSTEM



the performance of the entire system, the most appropriate stage would be as close as possible to the final output of the system, i.e., the Head-Up Display. On the other hand, it may be possible to identify inadequacies in various subsystems if target state vectors are evaluated at appropriate states closer to the sensor inputs.

A second interpretation feature concerns the coordinate frame in which the "true" target state vectors are to be compared with target state vectors from the director system. There are at least three commonly used reference frames: (1) an inertial frame, fixed in the earth or in a local air mass, in which the coordinate directions are north(x), east(y), and down(z); (2) an aircraft body frame, in which the coordinate directions are forward along the aircraft axis(x)*, out the right wing(y), and the right-hand cross product of these directions(z); and (3) a line-of-sight frame, in which the coordinate directions are along the line-of-sight to the target(x), toward the front of the aircraft(y), and the right-hand cross product of these directions(z). The director system, of course, always considers the target state vectors relative to the attacker, and thus the evaluation methods to be devised should also refer to the attacker as the origin. In general, if the comparison is to be made at a stage close to the target sensor inputs of the director system, then a line-of-sight frame is probably indicated, and if this comparison is to be made at a stage close to the final output, then the attacker's body frame is probably appropriate. If only the magnitudes of the

* A "velocity" frame, in which the x direction is along the total velocity, is also used.

target state vectors are to be compared, then the orientation of the coordinate frame need not be specified.

A third interpretation feature concerns the independence of the evaluation methods. Completely independent methods would not make use of any of the sensor inputs or computational algorithms used by the director system. However, such methods, although possible in principle, would generally be difficult or expensive to implement, and thus methods which are not completely independent of the director system must be considered in practice.

A fourth and final interpretation feature concerns the relative precision of the target state vectors obtained from the director system and the "true" vectors obtained by the methods to be devised. Thus the precision required of the "true" vectors, in order that they may be considered adequate for the evaluation of the director target state vectors, is not specified.

The above considerations, which may lead to significantly different interpretations of the target state measurement task, were discussed with representatives from AFAL/RWT and from Honeywell, and some decisions were made. First, it was decided that the director system target state vectors would be evaluated at a stage close to the final output. This decision allows an assessment of the effects of error propagation and accumulation, although information on the effect of any particular stage may be lost. In a second and related decision it was determined that the target state vectors would be obtained in the attacker's body frame, since this is the coordinate frame in which these vectors are most readily available at states close to the director system output.

A third decision determined that only instrumentation planned or readily available for tests of the director system would be used to devise independent methods for the evaluation of target state vectors. Table 6 shows the real and test inputs which will be available for testing the director system. The real inputs could be used in an actual combat encounter, whereas the test inputs are available for test purposes only. Note that tracking of the target and attacker from the ground or from an observation aircraft is not available but that various test inputs transmitted from the target to the attacker are available. The test data must be used to obtain (for the evaluation methods to be devised) as much independence from the director system as possible.

In a fourth decision it was determined that unproven or unstable analysis techniques would be avoided. It is possible, for example, to obtain velocity from acceleration if the initial integration time is known, and it may be possible to identify this initial time as the time at which integration must begin to obtain known roll, pitch, and yaw angles from known roll, pitch, and yaw rates. However, this is an apparently unproven technique which may require an unacceptable amount of computer time or memory if it were implemented on the attacker's computer. Differentiation to obtain velocity from position or acceleration from velocity is an example of a generally unstable technique (in the sense that initial disturbances are accentuated), although appropriate smoothing or filtering techniques may make this procedure acceptable.

TABLE 6

REAL AND TEST INPUTS
TO THE DIRECTOR SYSTEM

REAL INPUTS	TEST INPUTS
ASCOT	SSC
RADAR	MHC
IRST	DME
HMS	HARS
ISA	TELEMETRY FROM TARGET
ADC	ISA
TIME CODE	HARS
MODE CONTROL	ADC

ASCOT - ADAPTIVE SCAN OPTICAL TRACKER

IRST - INFRARED SEARCH AND TRACK

HMS - HELMET MOUNTED SIGHT

ISA - INERTIAL SENSOR ASSEMBLY

ADC - AIR DATA COMPUTER

DME - DISTANCE MEASURING EQUIPMENT

HARS - HEADING ATTITUDE REFERENCE SYSTEM

SSC - SOLID STATE CAMERA

MHC - MINI HUD CAMERA

HUD - HEADS-UP DISPLAY

Finally, it was decided to use Monte Carlo methods to obtain error probability distributions for the target state vectors calculated according to the interpretations and decisions discussed above. These distribtuions could then be compared with known values for the anticipated accuracy of target state vectors from the director system. Anticipated accuracy is, at present, available only for the magnitude of the acceleration of the target relative to the attacker: a standard deviation of about 1g. The method used to calculate an independent acceleration and an associated error distribution is determined by the decisions and interpretations discussed above, and it involves the use of simulated ISA and HARS data from both target and attacker. The mean of the error distribution for this method, in the most realistic simulation, is about 1.4 ft/sec^2-much less than 1g. Thus, if the various distributions assumed for the Monte Carlo simulation are appropriately conservative, and if the errors in the evaluation method are sufficiently uncorrelated with the director system, then it may be reasonable to conclude that this evaluation method is adequate for the verification of target acceleration obtained from the director system. Since anticipated accuracy figures for the director system are not available for velocity and position, no comparisons can be made for these quantities.

Note that no attempt is being made, in the current task, to locate the source of any discrepancies which may be observed between the target state vectors obtained from the director

system and the "true" vectors obtained by independent evaluation methods. An ability to identify the source of such discrepancies may form an important extension to the target state measurement task.

3.2.2 Formulation of the Monte Carlo Study

The objective of the Monte Carlo study is to obtain probability distributions for the errors in target acceleration, velocity, and position for certain evaluation methods. These methods are determined by the decisions made concerning the interpretation of the task objective, as discussed above. Referring again to Table 6, none of the real inputs should (ideally) be used for any evaluation method, but the ISA and ADC inputs, since they refer only to the attacker, may be used if necessary. The ISA units on both target and attacker are Honeywell H478 inertial sensor assemblies and are strapped-down accelerometer and gyro packages whose outputs include acceleration components and roll, pitch, and yaw rates. True air speed and angle of attack are available from the Air Data Computer on each aircraft. The Heading Attitude Reference System is part of the Stable Coordinate Reference Group (SCRGS) which in turn is part of the MA-1 Fire Control System on both target and attacker. (However, HARS is not a real input to the director system). The HARS outputs are sines and cosines of roll, pitch, and yaw angles. The Distance Measuring Equipment consists of a transmitter on the attacker and a transponder (remitter) on the target. The SSC is a Fairchild CCD solid state electronic gunsight camera

mounted with (but independent of) ASCOT on the attacker. The KB-25 Mini HUD (Head-Up Display) camera records images seen by the pilot on and through the HUD, including images of the target. Although the resolution of this camera may be high, the number of frames available during a simulated combat encounter is probably inadequate, and thus a decision was made to use only the SSC to simulate line-of-sight angle data.

Following the decisions discussed above and in the introduction, the acceleration of the target relative to the attacker is available only using ISA and HARS data from both target and attacker. Similarly, the velocity of the target relative to the attacker is available only from ADC and HARS data from both target and attacker. Finally, the position of the target relative to the attacker is available only from DME and SSC data. If the components rather than the magnitudes of the above vectors are to be obtained, then the appropriate coordinate system is the attacker's body frame.

The kinematics of the above methods may be expressed as follows. Let $u^f_{x/y}$ represent the vector u of the object x relative to the object y in the reference frame f; e.g., $v^a_{t/a}$ is the velocity of the target relative to the attacker in the attacker's body frame. Let $T^e_s\ (q_x)$ represent a coordinate transformation from the initial frame s to the final frame e as a function of the quantities q for the object x; e.g., T^a_i $(\phi_a, \theta_a, \psi_a)$ is a transformation from an inertial to a body frame as a function of the roll(ϕ), pitch (θ), and yaw (ψ) angles

of the attacker. The target state vectors to be obtained for comparison with the same vectors from the director system are $a^a_{t/a}$, $v^a_{t/a}$, and $p^a_{t/a}$:

$$a^a_{t/a} = T^a_i(\phi_a, \theta_a, \psi_a)T^i_t(\phi_t, \theta_t, \psi_t)a^t_{t/i} - a^a_{a/i} \tag{21}$$

$$v^a_{t/a} = T^a_i(\phi_a, \theta_a, \psi_a)T^i_t(\phi_t, \theta_t, \psi_t)T^t_v(\alpha_t)v^v_{t/i} - T^a_v(\alpha_a)v^v_{a/i} \tag{22}$$

$$p^a_{t/a} = T^a_l(\varepsilon_a, \eta_a)p^l_{t/a} \tag{23}$$

Here a, v, and p are acceleration, velocity, and position vectors; i, v, and l are inertial, velocity, and line-of-sight frames; t and a refer to target and attacker or to the corresponding body frame; ϕ_x, θ_x, and ψ_x are roll, pitch, and yaw angles for either target or attacker (for x-a or t), and ε_a and η_a are the elevation and azimuth of the target relative to the attacker. The appropriate transformation matrices are

$$T^x_i(\phi_x, \theta_x, \psi_x) = [T^i_x(\phi_x, \theta_x, \psi_x)]'$$

$$= \begin{pmatrix} C\psi_x C\theta_x & S\psi_x C\theta_x & -S\theta_x \\ -S\psi_x C\phi_x + C\psi_x S\theta_x S\phi_x & C\psi_x C\phi_x + S\psi_x S\theta_x S\phi_x & C\theta_x S\phi_x \\ S\psi_x S\phi_x + C\psi_x S\theta_x C\phi_x & -C\psi_x S\phi_x + S\psi_x S\theta_x C\phi_x & C\theta_x C\phi_x \end{pmatrix} \tag{24}$$

$$T^x_v(\alpha_x) = \begin{pmatrix} C\alpha_x & 0 & -S\alpha_x \\ 0 & 1 & 0 \\ S\alpha_x & 0 & C\alpha_x \end{pmatrix} \tag{25}$$

$$T_1^x(\varepsilon_x,\eta_x) = \begin{pmatrix} C\varepsilon_x C\eta_x & -S\eta_x & S\eta_x\ C\eta_x \\ C\varepsilon_x S\eta_x & C\eta_x & S\varepsilon_x\ S\eta_x \\ -S\varepsilon_x & 0 & C\varepsilon_x \end{pmatrix} \tag{26}$$

where C and S represent sin and cos and ' indicates matrix transposition. Diagrams of the methods selected for the independent evaluation of target state vectors are shown in Figures 6, 7, and 8.

In addition to the kinematic expressions, probability distributions for the occurrence of the various input quantities are required, as well as probability distributions for the measurement errors in these quantities. Tables 7 through 10 show these input quantities and their estimated occurrence and accuracy distributions. Two occurrence distributions for the ISA acceleration components (i.e., the components of $a^t_{t/i}$ or $a^a_{a/i}$) were tested. The first is a uniform distribution over the range -9g to +9g for each component, and the second is a much more realistic distribution suggested by Capt. Jerry Kendrick of AFAL/RWT: the x and y components of acceleration (in the aircraft body frame) are normally distributed with zero mean and standard deviations of 1g and 0.25g, respectively, and the z component is distributed according to the log- normal distribution $(8-4e^{e/2})$g, where $\varepsilon \sim N(0,1)$. This distribution has a near cut-off at about 7.5g, a mean at about 3.5g, and a "tail" which extends to lower accelerations. Error distributions for the ISA acceleration components were obtained from a table of instrumentation signals for the F106 provided by Capt. Dave Schoor

FIGURE 6. DIAGRAM OF THE EVALUATION METHOD FOR THE ACCELERATION OF THE TARGET RELATIVE TO THE ATTACKER

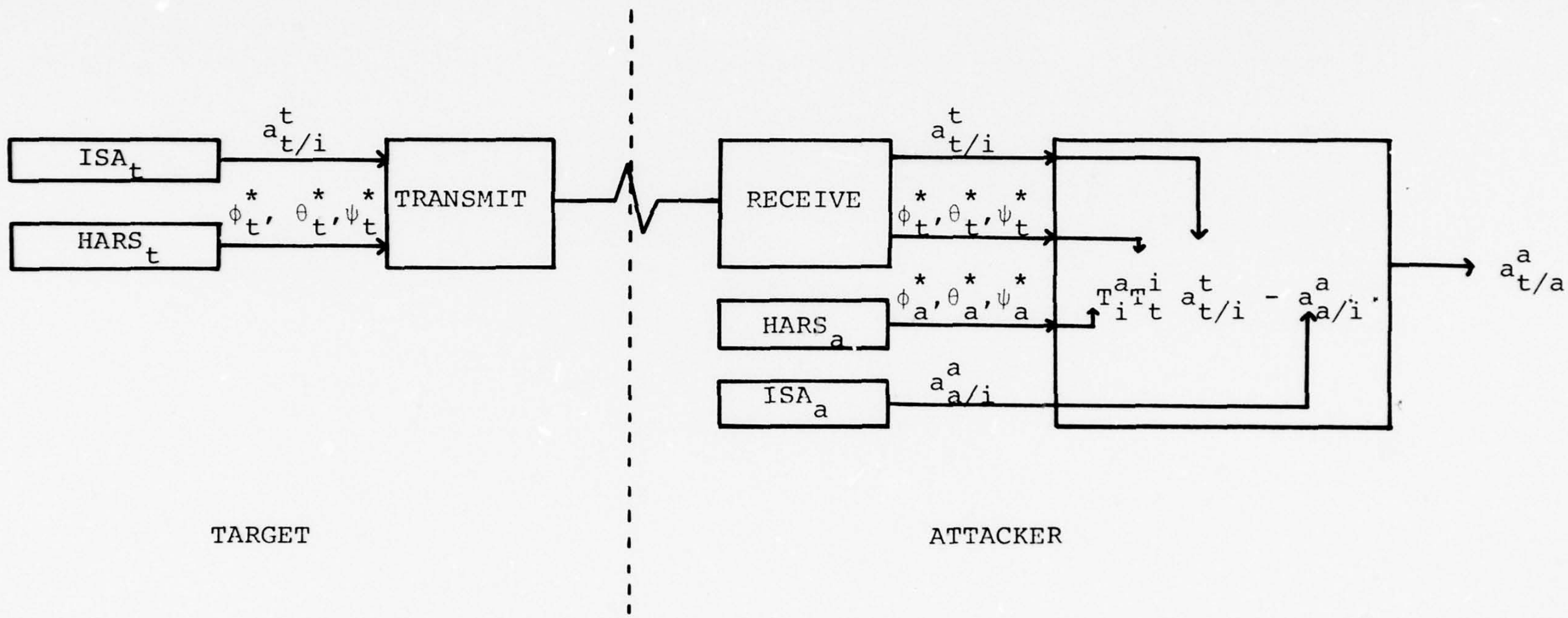


FIGURE 7. DIAGRAM OF THE EVALUATION METHOD FOR THE VELOCITY OF THE TARGET RELATIVE TO THE ATTACKER

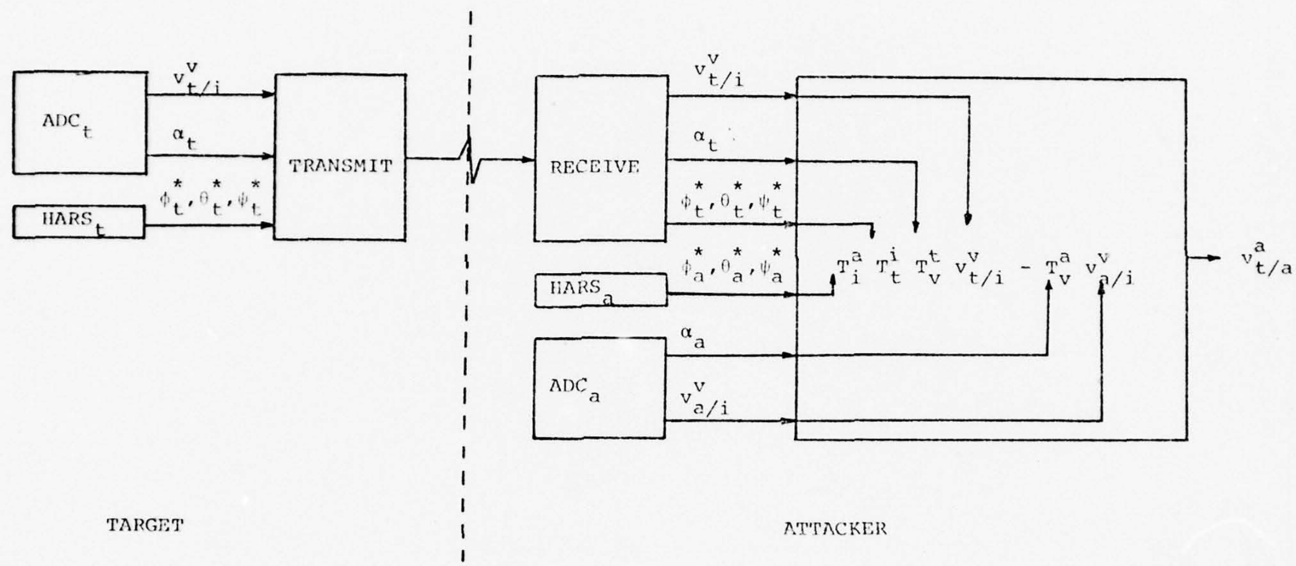


FIGURE 8. DIAGRAM OF THE EVALUATION METHOD FOR THE POSITION OF THE TARGET RELATIVE TO THE ATTACKER

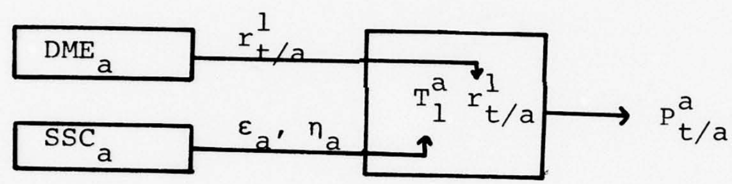


TABLE 7

OCCURRENCE AND ACCURACY DISTRIBUTIONS FOR THE ISA ACCELERATION COMPONENTS

	OCCURRENCE	ACCURACY
BROAD DISTRIBUTION	U(-9g, +9g)	N(0, 0.5 ft/sec^2)
NARROW DISTRIBUTION	x: N(0, 1g) y: N(0, 0.25g) z: $a_z = 8 - 4e^{\varepsilon/2}$ $\varepsilon \sim N(0,1)$	(above)

TABLE 8

OCCURRENCE AND ACCURACY DISTRIBUTIONS FOR THE HARS ROLL, PITCH, AND YAW ANGLES

		OCCURRENCE	ACCURACY
BROAD DISTRIBUTION	ϕ:	U(−180°, +180°)	N(0, 0.005)
	θ:	U(−90°, +90°)	for sin or cos of any angle
	ψ:	U(−180°, +180°)	
NARROW DISTRIBUTION	ϕ:	U(−60° → 90°, +60° → +90°)	
	θ:	U(−30° → +45° for 75% of pdf area)	(above)
	ψ:	U(−180°, +180°)	

TABLE 9

OCCURRENCE AND ACCURACY DISTRUBITONS FOR
ADC TRUE AIR SPEED ANGLE OF ATTACK

	OCCURRENCE	ACCURACY
$V^{V}_{x/i}$	U(200, 600 Knots)	N(0, 4 Knots)
α	U(-5, +20°)	N(0, 0.1°)

TABLE 10

OCCURRENCE AND ACCURACY DISTRIBUTIONS FOR DME RANGE AND FOR SSC AZIMUTH AND ELEVATION ANGLES

	OCCURRENCE	ACCURACY
$P^{1}_{t/a}$	U(0, 2500 ft)	N(0, 4 ft)
η, ε	U(-15°, +15°)	N(0, 0.07°)

at Tyndall AFB. Roll, pitch, and yaw ranges and measurement error distributions were also obtained from this table. Two different occurrence distributions were tested: uniform distributions over the permissible ranges and a more restricted piecewise uniform distribution suggested by Maj. Stewart Cranston at Tyndall AFB. Measurement errors for computed air speed and for the angle of attack were obtained from the table for the F106 mentioned above, and the occurrence distributions for these quantities were suggested by Capt. Jerry Kendrick (the side-slip angle was assumed to be zero). The occurrence and error distributions for the DME were suggested by Dr. James Yi at Honeywell. Finally, the required occurrence and accuracy distributions for the azimuth and elevation angles from the SSC were estimated from information provided by Capt. H. Woodruff and Mr. Jerry Watson of AFAL/RWT.

The procedure for the Monte Carlo calculations will now be described. Values for the components of $a^t_{t/i}$, $a^a_{a/i}$, $v^v_{t/i}$, $v^v_{a/i}$, and for the angles ϕ_a, θ_a, ψ_a, ϕ_t, θ_t, ψ_t, α_t, ε_a, and η_a were selected, independently, from the occurrence distributions given in Tables 7 through 10. Next measurement errors for these quantities were selected from the accuracy distributions given in these tables. The final vectors, $a^a_{t/a}$, $v^u_{t/a}$, or $p^a_{t/a}$, were then calculated according to Eqs. (21) through (26) both with and without the addition of the selected measurement errors to the selected components and angles. The resulting vectors were subtracted, and the entire procedure was repeated for 300 selection sets for each vector whose error distribution was to be

determined. Finally, probability distributions of the differences between the square root of the sum of the squares of the components (for vector magnitude), were plotted. The final result was a set of simulated error distributions for the evaluation methods defined by Eqs. (21) through (26) and diagrammed in Figures 6, 7, and 8. These evaluation methods are proposed as independent methods for evaluating the accuracy of target position, velocity, and acceleration vectors (relative to the attacker in the attacker's body frame) obtained from the director system.

3.2.3 Results of the Monte Carlo Study

Error distributions calculated according to the Monte Carlo procedure described above are given for acceleration in Figures 9 through 11. These distributions refer to the error in the acceleration of the target relative to the attacker in the attacker's body frame when this acceleration is obtained using ISA and HARS data from both target and attacker. Figure 9 shows the magnitude of this acceleration error calculated using the most realistic occurrence distributions for the ISA acceleration components and HARS angles (log-normal $(a^{x}_{x/i})_{z}$, piecewise uniform ϕ and θ). The mean of this distribution is about 1.4 ft/sec^2 - much less than the 1g (32 ft/sec^2) anticipated accuracy of the magnitude of the acceleration of the target relative to the attacker from the director system. In addition, Figure 9 shows that the probability that the ISA-HARS evaluation method will approach an error of 1g is very small, since the "tail" of the error distribution is already small at about 0.1g.

Figure 9. Error Distribution of the Magnitude of the Acceleration of the Target Relative to the Attacker Obtained Using Simulated ISA and HARS Data

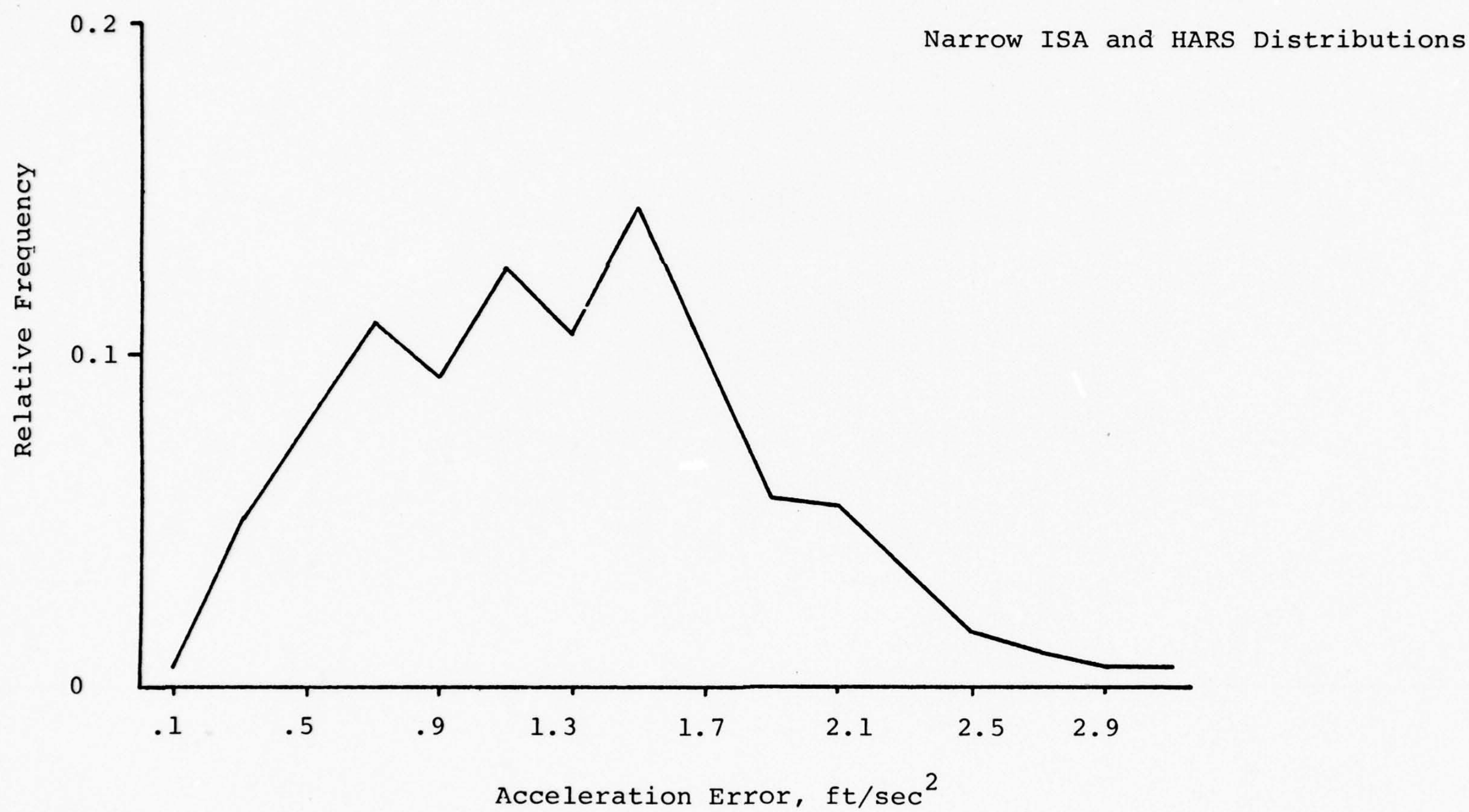


Figure 10. Error Distribution of the Magnitude of the Acceleration of the Target Relative to the Attacker Obtained Using Simulated ISA and HARS Data

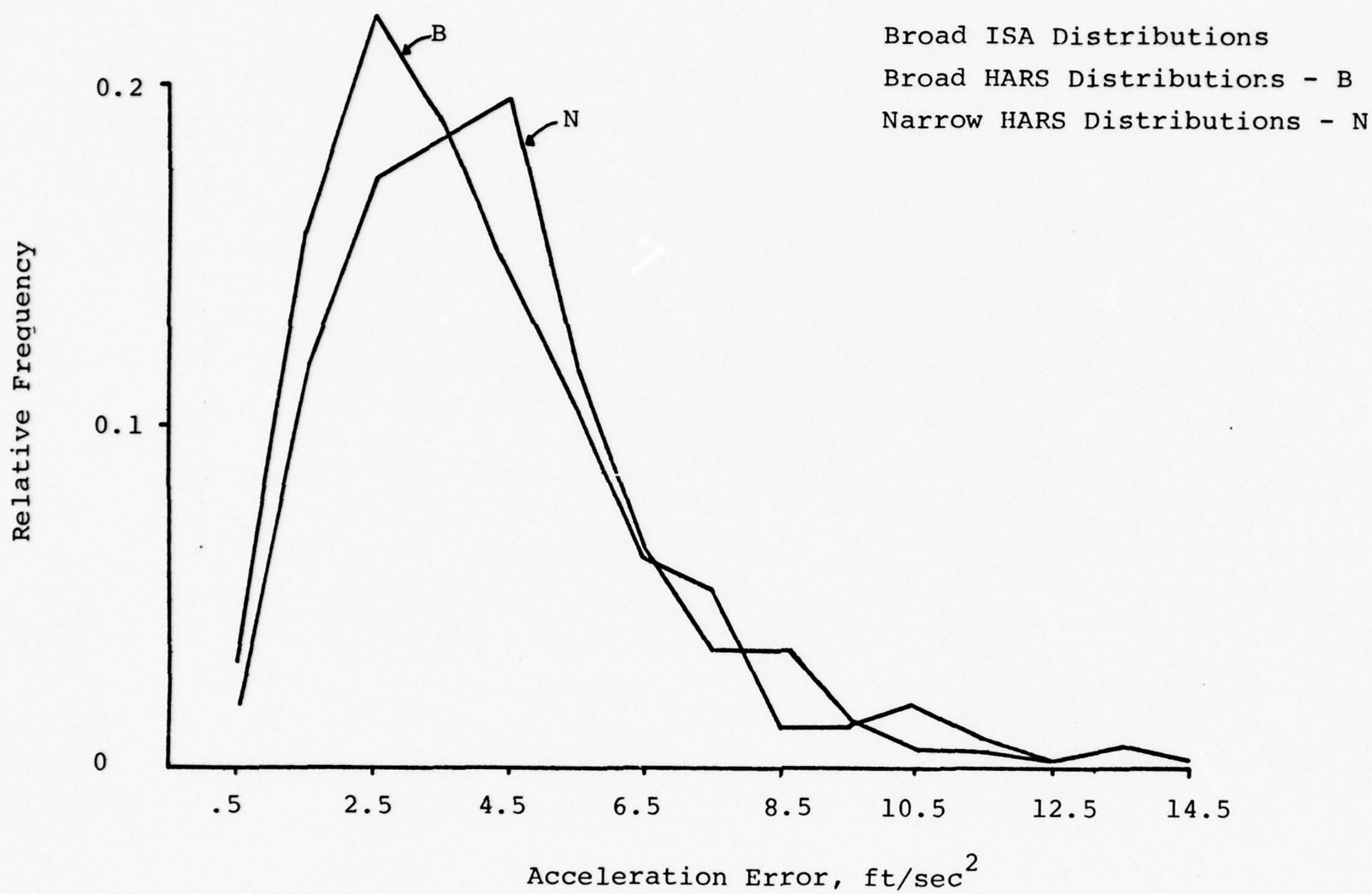
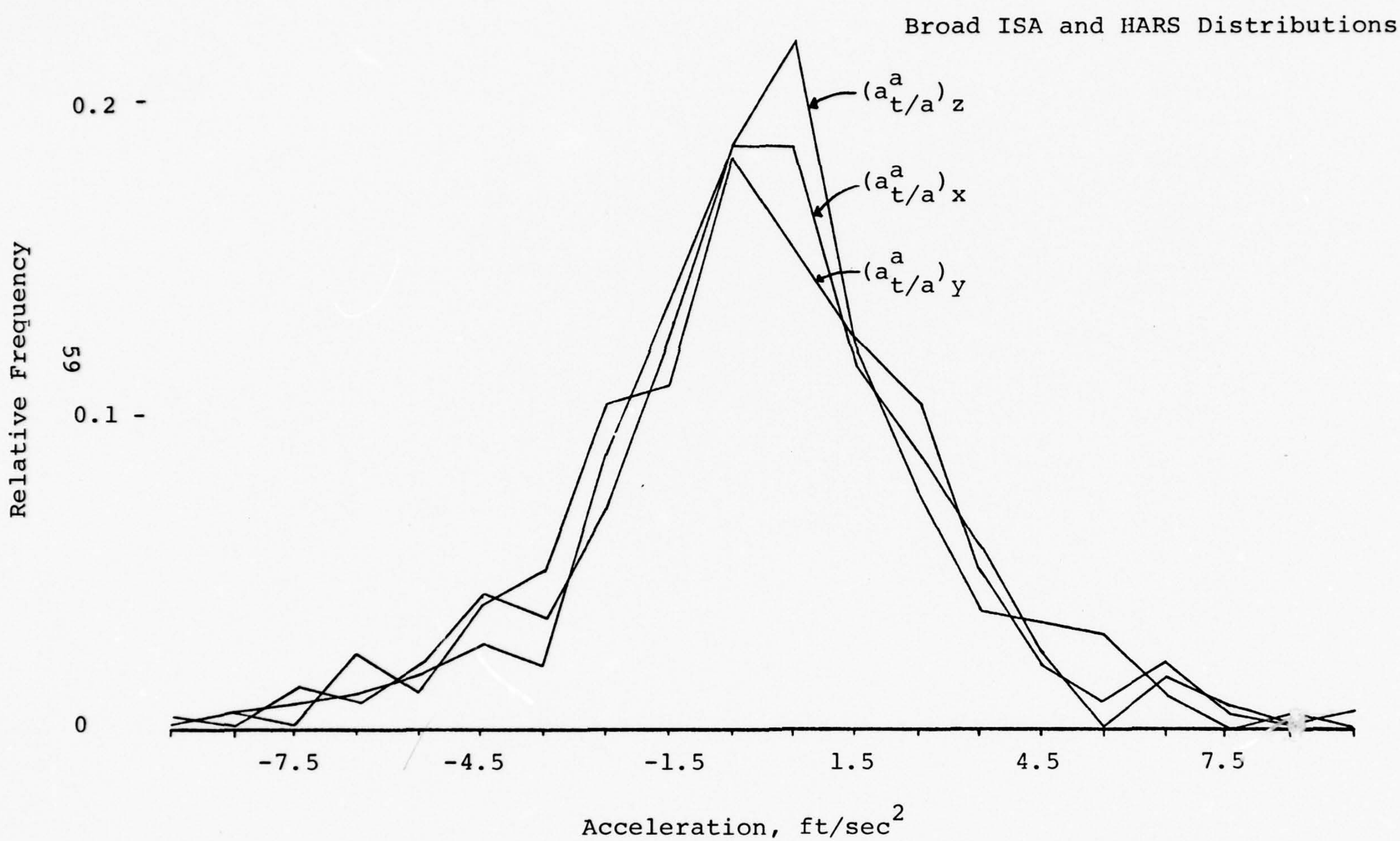


Figure 11. Error Distribution of Components of the Acceleration of the Target Relative to the Attacker in the Attacker's Body Frame Obtained Using Simulated ISA and HARS Data



The error distributions for acceleration magnitude shown in Figure 10 are calculated for uniform occurrence distributions of the ISA acceleration components (from -9 to +9g.), and from uniform or piecewise uniform occurrence distributions for ϕ, θ, and ψ. These error distributions have means considerably larger (about 3 ft/sec^2) than the distribution of Figure 9 because of the much greater (and probably much less realistic) probability of large acceleration. However, the use of the more restricted piecewise uniform distribution for roll and pitch angles seems to have only a minor effect on the final error distributions.

Error distributions for the x, y, and z components of the acceleration of the target relative to the attacker in the attacker's body frame are shown in Figure 11. These distributions are roughly the same since the occurrence and error distributions used in their deviation were identical (uniform distributions from -9 to 9g for the ISA acceleration components and uniform distributions for ϕ, θ, and ψ over their allowed ranges). As in Figures 9 and 10, the probability that any error in acceleration will exceed 1g is very small.

Figures 12 and 13 show error distributions for the magnitude of the velocity and position vectors of the target relative to the attacker. The restricted piecewise uniform occurrence distributions for ϕ, θ, and ψ were used for the velocity calculations. The mean error in velocity is about 8 knots and the mean error in position is about 4 ft.; both distributions are skewed toward larger magnitudes. Although these distributions

Figure 12. Error Distribution of the Magnitude of the Velocity of the Target Relative to the Attacker Obtained Using Simulated ADC and HARS Data

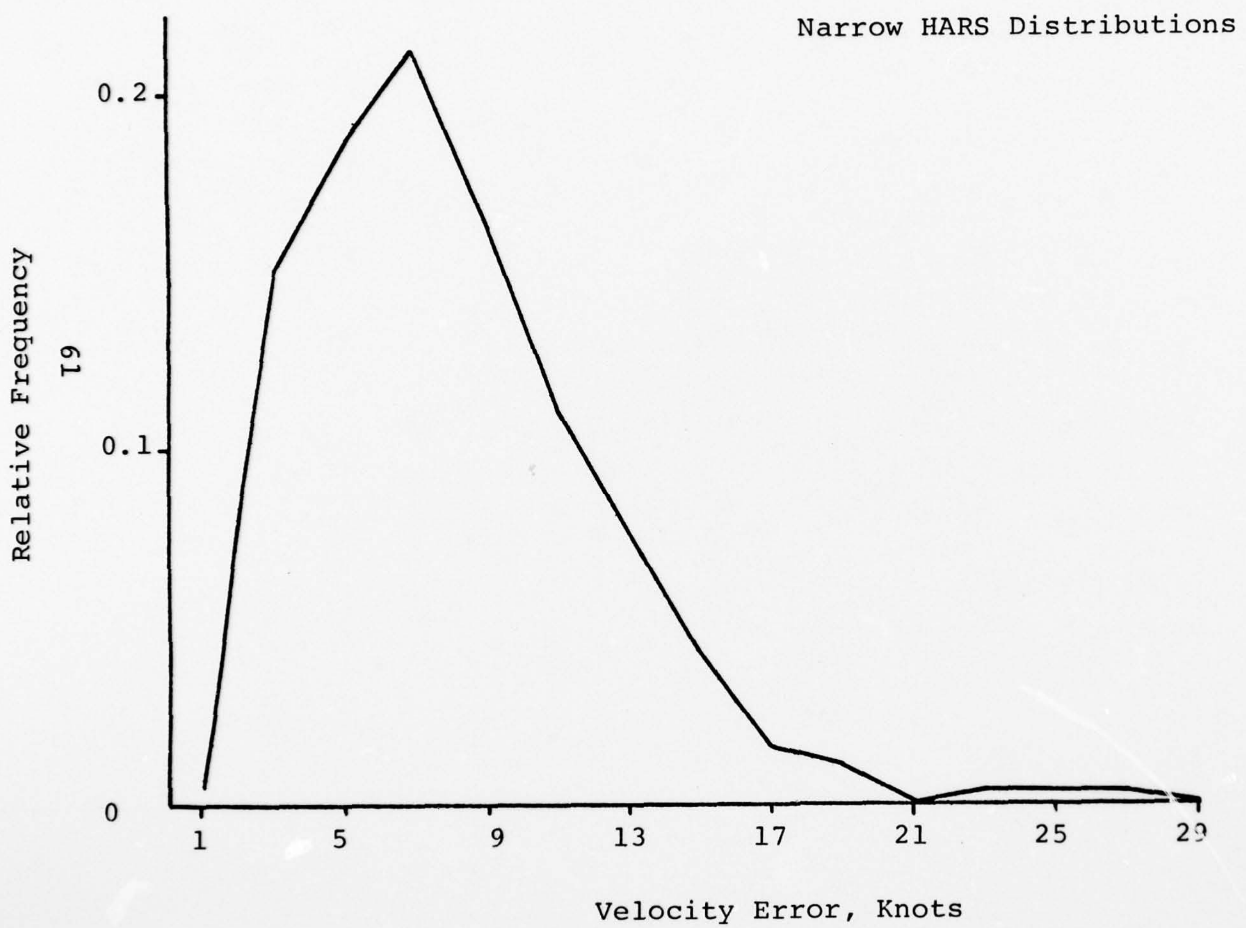
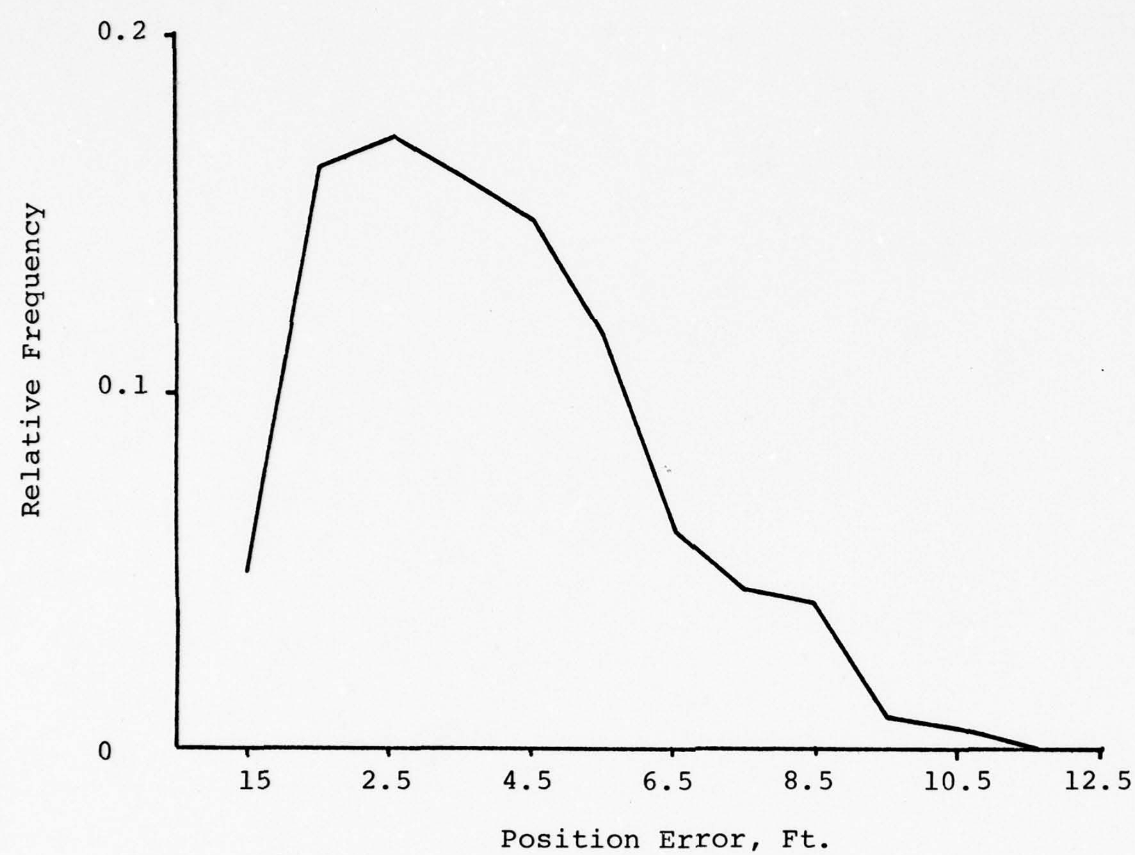


Figure 13. Error Distribution of the Magnitude of the Position of the Target Relative to the Attacker obtained using Simulated DME and SSC Data.



provide some information on the accuracy of the ADC-HARS method for velocity and the DME-SSC method for position, the anticipated accuracy of the director system for these vectors is not known at present, and thus the adequacy of these methods for the evaluation of the director system cannot be quantitatively assessed.

3.2.4 Suggestions for Future Study

As discussed in the introduction, the target state measurement task has been interpreted so that it may be said to be complete when independent methods are devised and shown to be adequate for the evaluation of target state vectors obtained from the director system. It is quite likely that unacceptable discrepancies will be found between the target state vectors from the director system and the "true" vectors from the independent evaluation methods. In the current task, no attempt is made to locate the source of these anticipated discrepancies; however, such an attempt is believed to be essential for an adequate evaluation of the complete director system.

There are at least three apparent approaches to the problem of "tracing back" observed errors in the director target state vectors. First, it may be possible to perform a sensitivity study to find the stages in the director system at which major discrepancies originate. Second, the target state measurement task, as interpreted here, might be carried out at several different stages in the director system (from the sensor inputs to the HUD output). Finally, a detailed error analysis of the entire director system, including input, processing, and output

errors, could be made. Results from any of these approaches could lead to significant performance improvements in the director system.

3.3 IDENTIFY AND CORRECT RADAR LAG PROBLEM

3.3.1 Introduction

3.3.1.1 Background

For proposed weapons systems to be effective, accurate knowledge of the target state is needed. The EXPO V and Fire/Fly programs pointed out the need for the results of improved trackers and fire control algorithms. To this end, the AFAL initiated the Gunsight Evaluation Program (Sight Eval.) to evaluate the performance of existing fire control systems. This program, conducted at Tyndall AFB using an instrumented F-106 which tracked towed and radio-controlled targets and made both live and dry firing missions, produced data which could be used for the analysis of existing fire control systems.

An important part of the instrumentation aboard the F-106 was the MA-1 radar, which measured range and range rate, elevation and elevation rate, azimuth and azimuth rate. It is the purpose of this report to analyze the performance of the MA-1 radar using the above data.

3.3.1.2 Motivation

The target state is estimated from measurements of range, azimuth, and elevation, along with the first and second derivatives of range, azimuth, and elevation. The derivatives are obtained from suitable algorithms using direct measurements of target range and angular position.

The MA-1 radar provides adequate range information, but its measurements of azimuth and elevation are

not sufficiently precise to be useful to hand off target position to an electro-optical sensor (ASCOT), whose field of view is much narrower than the MA-1, and which measures the angular position of the target with sufficient accuracy.

The major system integration problem with the above technique is that the two fields of view of the ASCOT are limited to $\pm$ 17 mrad and $\pm$ 43 mrad, whereas the MA-1 reports angular positions which are in error by as much as $\pm$ 50 mrad. It is the purpose of this analysis to improve the radar estimates of the angular position such they have a high probability of lying within the $\pm$ 17 mrad window of the ASCOT.

3.3.1.3 Scope and Objectives of the Problem

There are two different methods which can be used to do the above. The first, called the hardware approach, is to modify the design, mounting, or calibration of the MA-1 such that the desired degree of accuracy can be achieved. The second, a software approach, is to look for a suitable algorithm which will produce correction terms such that the revised radar estimates lie within the allowable error tolerances.

The principal drawback to the hardware approach is the lack of access to the design and installation data on the modified MA-1 system and to the schedules of the alignment and calibration procedures performed on the system during the Sight Eval tests.

The second approach is a possible alternative. The ROLM 16/64, a rugged, high-speed, airborne computer is a component of the proposed fire control system, and could conceivably contain provisions for correcting the angular data provided by the MA-1. It is along these lines that this study has been directed.

Two distinct efforts related to a software approach were pursued. One effort involved a relatively straightforward graphical, curve-fit attack attempting to relate tracking error to tracking rate functions. The second effort involved an analysis of the relative angular rate of change of the Line of Sight (LOS) between the aircraft body axis and the target followed by a similar analysis of angular rates between aircraft and target relative to an inertial coordinate system.

3.3.2 Data Collection

3.3.2.1 General

The original purpose of the Sight Eval program was to test and evaluate fire control systems, especially the HLGS (Hot Line Gun Sight) and the DALCOS (Digital Advanced Lead Computing Optical Sight). As mentioned previously, the objective of this study is to determine the magnitude and causes of the angular position errors in the angular position measurements of the MA-1 Fire Control System, and to develop an algorithm to reduce these errors. Of primary concern is the difference between the "true" target position, determined from a 16 mm film record, and the position reported by the MA-1 radar.

The radar target position and rates are determined by resolvers mounted on the MA-1 antenna gimbals, and by interceptor rate gyros and radar range measurements.

The data available for this study consisted of analog signals recorded on magnetic tape and target angular position recorded on 16 mm film. After each mission, the magnetic tape and film data were converted to digital form and recorded on tape.

3.3.2.2 Description of Test Missions

Ninety-five live firing missions were planned and executed for the purpose of evaluating gunsight algorithms, but only 39 of these missions were recorded.

Figure 14 shows the relationship between target and attacker in a typical firing encounter. In one mission, the target was remotely piloted PQM-102. In the remaining missions, the targets were towed. The towed targets were either the 30 ft long fiberglass FIGAT or the 15 ft long aluminum DART.

The firing tests were conducted at different ranges, altitudes, angle-off at firing, and target acceleration. The target was in either a 1g, 2g, or 4g turn. The attacker coordinated his pursuit so that firing began at prescribed angles-off (10°, 20°, 30°, or more) and ranges (1000, 1500, 2000, or 3000 ft) at altitudes of 10,000, 15,000, and 20,000 ft. Figure 15 is a matrix showing the combinations of these variables which were used in the tests.

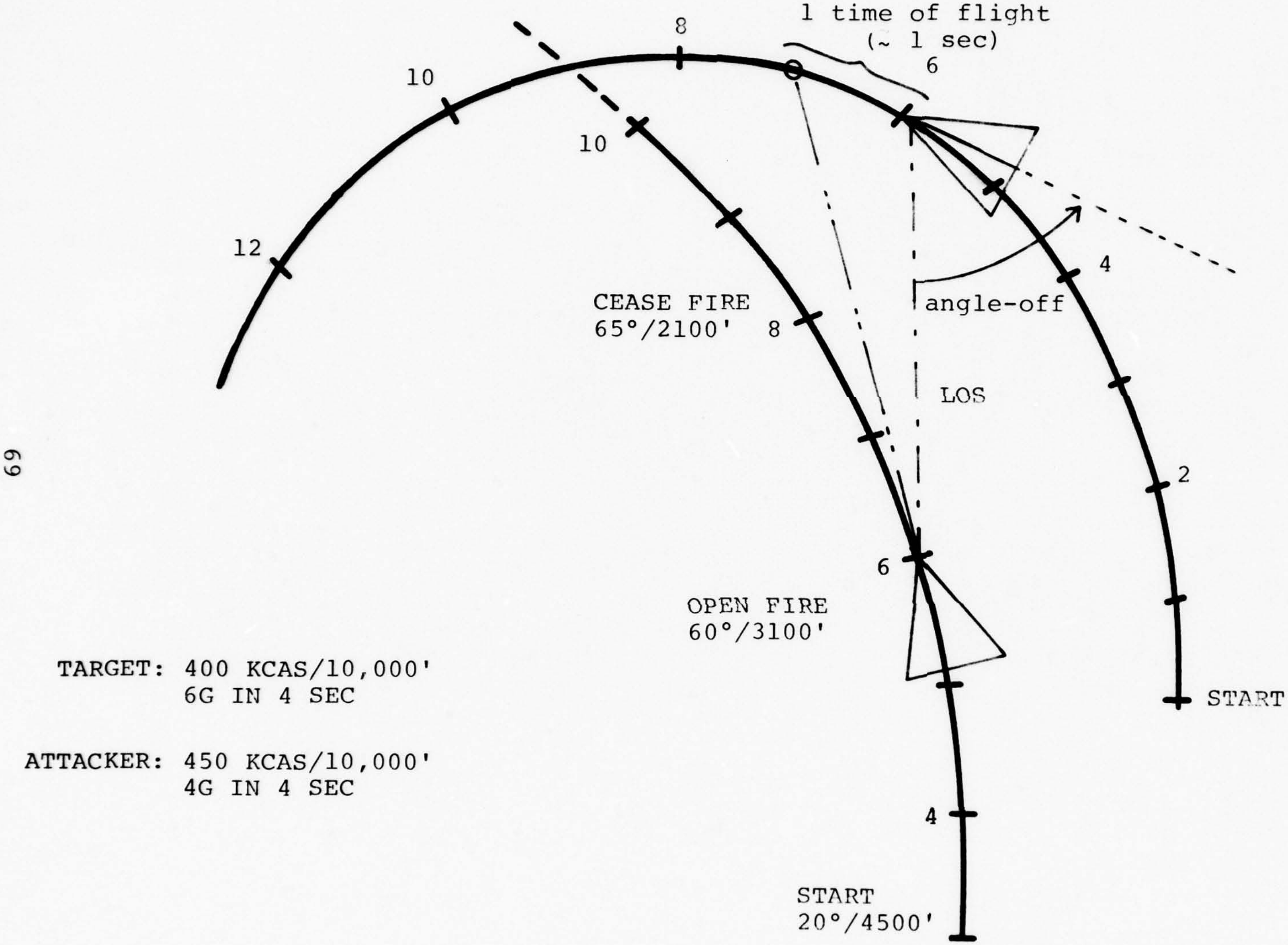


Figure 14 Firing Profile for Actual Mission With a Drone Target (PQM-102)

Altitude	Range (1000 Feet)	1g BANNER				2g FIGAT				4g DART			
	Acceleration Target / Angle Off at Firing (Degrees)	10	20	30	>30	10	20	30	>30	10	20	30	>30
10000 ft.	1.0					X	X	X	X	X	X	X	
	1.5				X		X	X	X	X	X		X
	2.0				X		X	X	X	X		X	X
	3.0					X	X	X	X		X		
15000 ft.	1.0		X		X	X			X				X
	1.5			X		X	X	X	X	X		X	
	2.0					X	X	X	X		X		
	3.0		X			X			X	X			X
20000 ft.	1.0				X	X	X	X					
	1.5	X				X	X		X	X			X
	2.0			X		X		X					
	3.0				X	X	X	X	X				

Figure 15 Test Matrix

3.3.2.3 Instrumentation and Recorded Data

A block diagram showing the transfer of analog data from the sensors to the recording tape is shown in Figure 16. The ISA (H 478 Inertial Sensor Assembly) measures ownship acceleration in rectangular components. The IRP (Inertial Reference Package) gives ownship pitch, yaw, and roll rates. The SCRG (Stable Coordinate Reference Group) - AHARS (Altitude Heading and Reference System) gives ownship pitch, yaw, and roll.

Radar, operating in conjunction with the CADC (Central Air Data Computer), gives range and range rate. The IR, and infra-red tracker, provides passive angular target direction for powered targets. These signals are input to an HCM-204 Fire Control Computer. The β -vane is a yaw attitude sensor.

All the signals described above are fed into a signal conditioner, whose function is to adjust the amplitude of the incoming analog signals to a 0-5 volt level, and to generate a Pulse Amplitude Modulated (PAM) train, which is an ordered sequence of all signals. The sets of data on the PAM train are separated by about 70 msec. The output of the signal conditioner is recorded on a Genesco tape recorder. The output of the IRIG - B Time Code Generator is recorded directly onto the tape, to the nearest thousandth of a second.

The conversion of data from the ISA and the MA-1 sensors into a PAM train by the signal conditioner is shown in Figure 17. The PAM train, together with ballistic

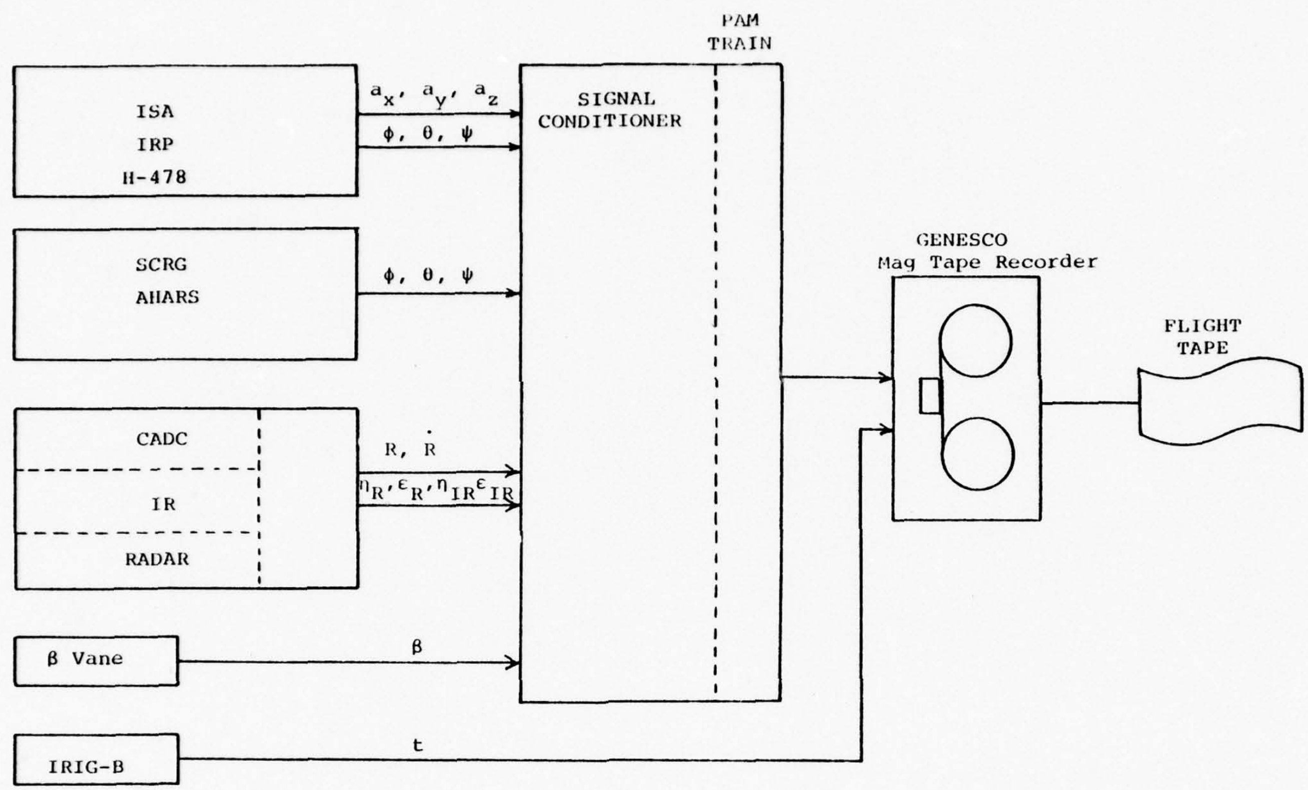


Figure 16 Electrical Analog Signals Recorded in Flight During Sight Eval

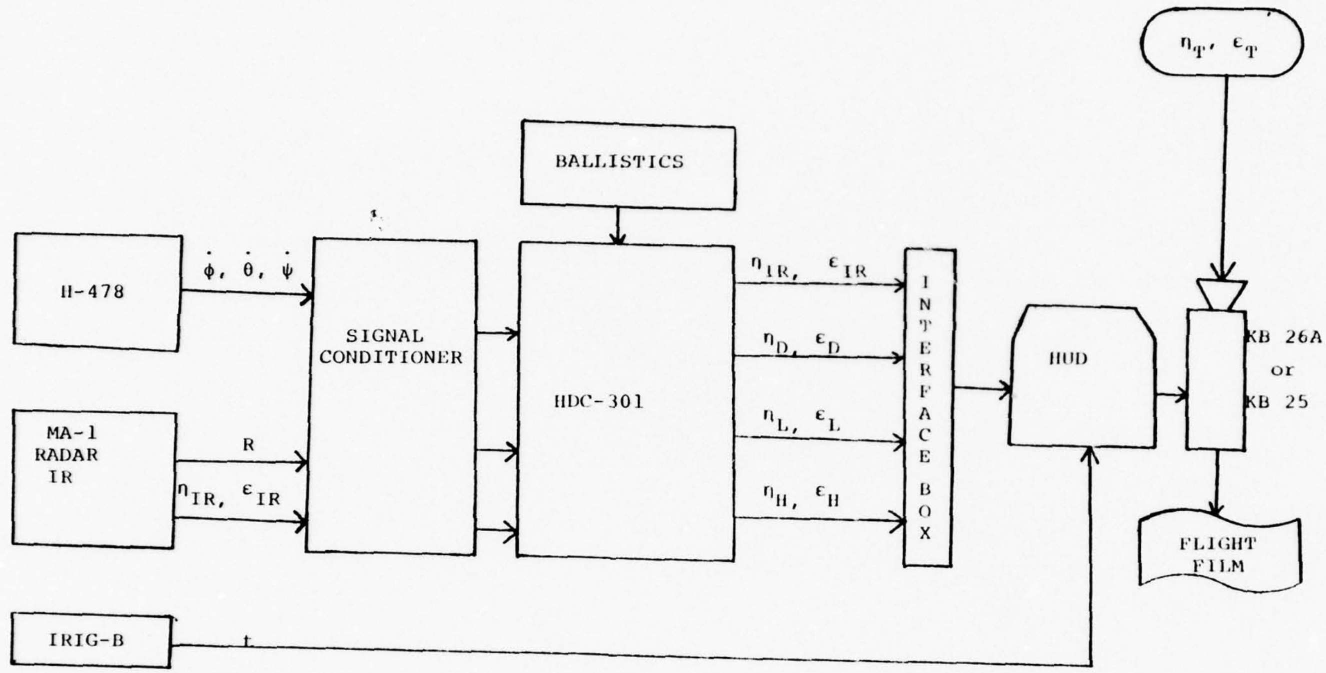


Figure 17 Signals Recorded on Flight Film

information for the type of projectile used, are fed into a HDC-301 computer. Using an algorithm peculiar to each of the gunsights to be compared, the computer determines the future azimuth and elevation of the target. An interface box converts these signals into a form such that they may be rendered as symbols on the HUD (Heads Up Display). They are simultaneously seen by the pilot and recorded on flight film by a KB 25 or KB 26A camera.

The conversion of the flight film and flight tape to digital form and the result recorded on tape in a form compatible with the CDC 6600 is shown in Figure 18. The empty box symbolizes unknown processing and transcribing operations which occurred in some cases. The result, the WPAFB tapes, were the raw material for this analysis.

3.3.3 Preliminary Data Evaluation and Processing

3.3.3.1 Description and Selection of Data

The information on the WPAFB tapes came from data recorded during test missions flown at Tyndall AFB during 1975 and 1976. Figure 19 lists the missions for which data were recorded and supplied to WPAFB. Each mission consisted of a number of passes, and only data from wet (firing) runs were recorded. A study of the data led to the classification of the missions into seven groups, two of which contain but one mission. Data from each pass were considered in the light of pilot comments. In some cases of obvious malfunction; e.g., no radar lock-on, the pass was disregarded.

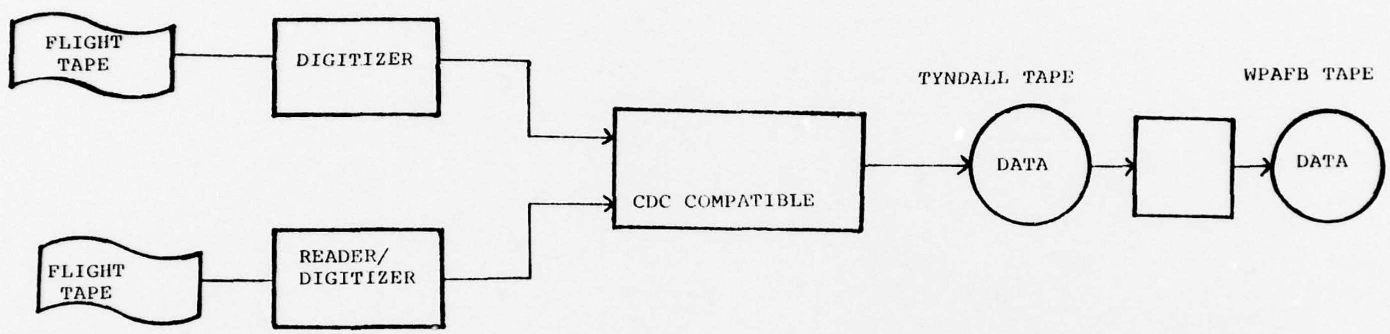
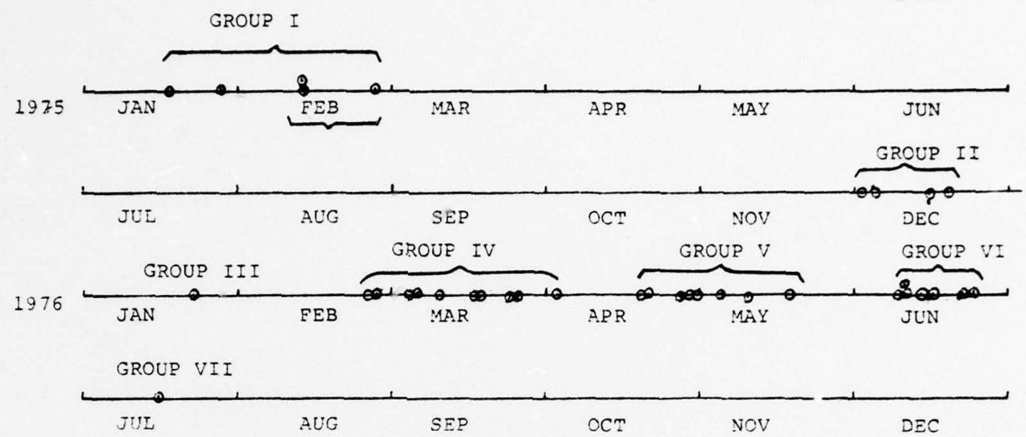


Figure 18 Processing From Flight Records to the WPAFB Data Tape



GROUP	MISSION DATE	ACTUAL DATE	TAPE	GROUP	MISSION DATE	ACTUAL DATE	TAPE
I	75017	17 JAN 75	1187	IV	76085	25 MAR 76	1183
	75027	27 JAN 75	"	(Cont.)	76092	01 APR 76	"
	95044	13 FEB 75	"		76093	02 APR 76	"
	75044	13 FEB 75	"				
	75058	27 FEB 75	"	V	76110	19 APR 76	197
					76111	20 APR 76	"
II	75336	02 DEC 75	719		76118	27 APR 76	"
	75338	04 DEC 75	1186		76119	28 APR 76	"
	75345	11 DEC 75	719		76121	30 APR 76	"
	75346	12 DEC 75	"		76126	05 MAY 76	301
	75349	15 DEC 75	"		76131	10 MAY 76	"
	75353	19 DEC 75	"		76139	18 MAY 76	"
III	76022	22 JAN 76	270	VI	76162(#1)	10 JUN 76	301
					76162(32)	10 JUN 76	1186
IV	76057	26 FEB 76	270		76163	11 JUN 76	301
	76058	27 FEB 76	"		76166	14 JUN 76	"
	76064	04 MAR 76	"		76167	15 JUN 76	1186
	76065	05 MAR 76	"		76174	22 JUN 76	"
	76070	10 MAR 76	"		76175	23 JUN 76	"
	76077	17 MAR 76	1183				
	76078	18 MAR 76	"	VII	76197	15 JUL 76	1186
	76083	23 MAR 76	"				

Figure 19 Recorded Missions

Each pass record consists of the PAM data plus signals from the time signal generator. Thirty-seven channels of data recorded on the WPABF tapes are listed in Table 11.

Data relevant to this analysis were the time, range, pitch rate, yaw rate, radar antenna azimuth, radar antenna elevation, film target position azimuth, and film target position elevation. When the validity of the data was in doubt, accelerometer, rate gyro, and heading information were examined as possible sources of error.

3.3.3.2 Evaluation and Preliminary Processing of Data

The film records were made on 16 mm film by a KB-25 or KB-26A camera operating at 24 frames per second. Air Force engineers have estimated a maximum error of 1 mrad due to film sensitivity. Alignment has reduced parallax to about the same magnitude. Because of the finite size of the target as seen on the film, a point on the target - usually the tail - was chosen as the target location. However, inconsistency in film reading practice could have led to variations of as much as 5 mrad in the target position.

At close range, an error of from 10 to 20 mrad could have been introduced due to differences between the point on the target selected from film reading and that which was most strongly reflecting the radar signal.

Film elevation and azimuth data and radar range data, elevation, and azimuth data were smoothed by means

TABLE 11

TAPE DATA VARIABLES

WORD NO. IN RECORD	VARIABLE	UNITS
1	IRIG TIME	MSEC
2	TRACKER MODE(0=Range, 1=IR)	————
3	MACH NUMBER	————
4	IMPACT TEMPERATURE	°R
5	STATIC PRESSURE	LES/FT^2
6	ANGLE OR ATTACK	RAD
7	RANGE	FT
8	RANGE RATE	FT/SEC
9	MANUAL RANGE	FT
10	TRIGGER #1 (1.= ON)	————
11	TRIGGER #2 (1.= ON)	————
12	RANGE ON TARGET (1.=LOCK)	————
13	DELAYED RANGE ON TET (1.=LOCK)	————
14	ROLL RATE	RAD/SEC
15	PITCH RATE	RAD/SEC
16	YAW RATE	RAD/SEC
17	X ACCELEROMETER	FT/SEC^2
18	Y ACCELEROMETER	FT/SEC^2
19	Z ACCELEROMETER	FT/SEC^2
20	SIDESLIP ANGLE	RAD
21	EVENT (1.= ON)	————
22	HEADING	RAD
23	ROLL	RAD
24	PITCH	RAD
25	RADAR ANTENNA AZIMUTH	RAD
26	RADAR ANTENNA ELEVATION	RAD
27	————————	————
28	————————	————
29	IR GIMBAL, AZIMUTH	RAD
30	IR GIMBAL, ELEVATION	RAD
31	AMMO TEMPERATURE (UNCALIB.)	
32	FILM TARGET POSITION, AZIMUTH	MRAD
33	FILM TARGET POSITION, ELEVATION	MRAD
34	FILM LCOS POS, AZIMUTH	MRAD
35	FILM LCOS POS, ELEVATION	MRAD
36	FILM HLES POS, AZIMUTH	MRAD
37	FILM HLES POS, ELEVATION	MRAD
38	FILM DESIGNATOR POS, AZIMUTH	MRAD
39	FILM DESIGNATOR POS, ELEVATION	MRAD
40-50	(BLANK)	————

of a polynomial function, although the raw film data were reasonably smooth.

The rms error between the radar range data and the fitting function was calculated for 90 passes and the results plotted in Figure 20. The rms variation in radar range lies between 12 and 53 ft (4 to 16 m) and shows no strong dependence on range. If one assumes that the maximum rms deviation of 53 ft corresponds to 3 σ, then the noise in the radar range measurements has a standard deviation of less than 6 m.

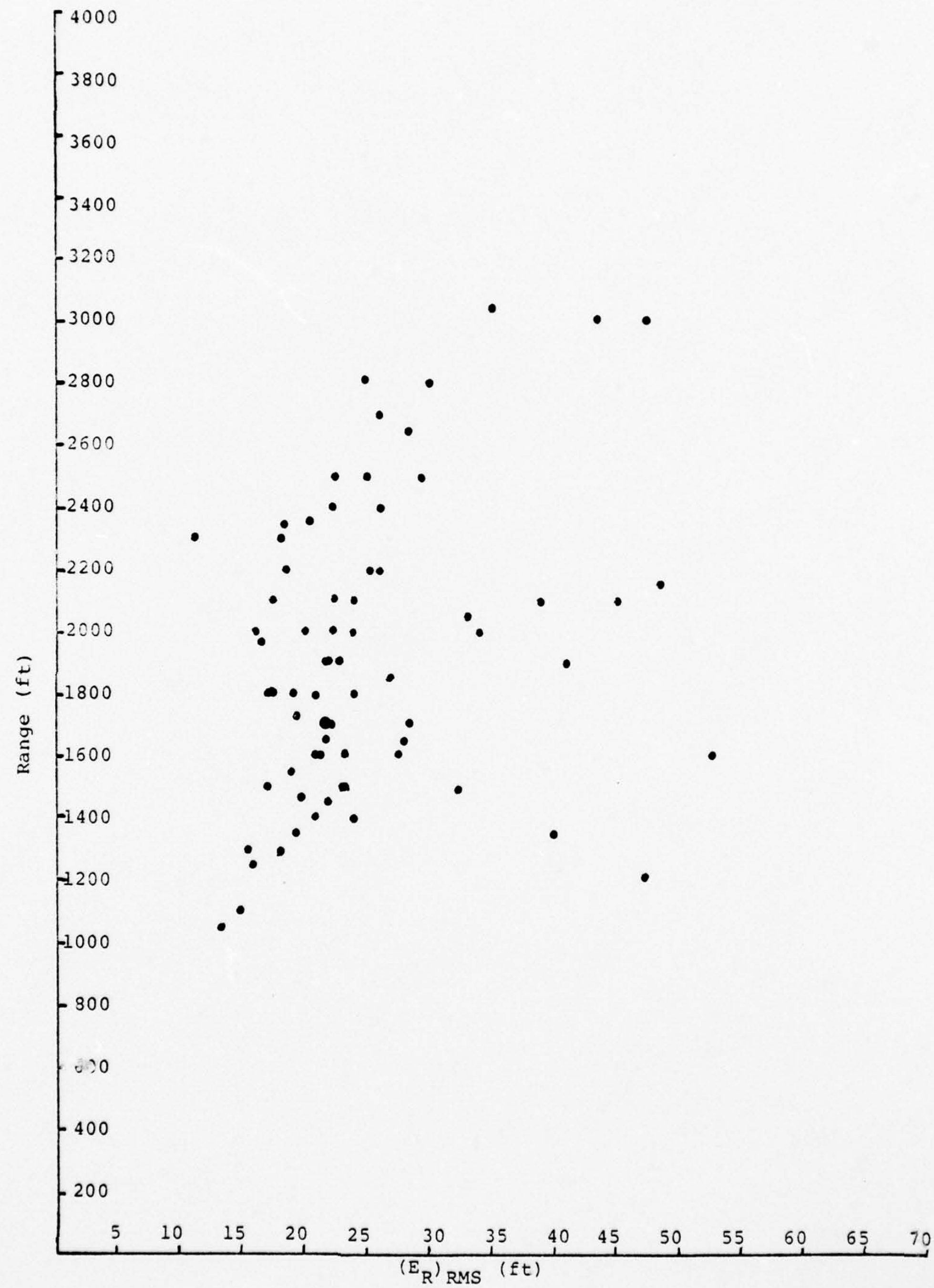


Figure 20 RMS Range Error

3.3.4 Analysis and Correction of Lag Problem

3.3.4.1 Linear Regression

Preliminary plots of radar and film data indicated that the radar data was usually lagging the film data. To use the radar to predict the target position at a future time point or to aim the ASCOT, it is necessary to compensate for the lag. The ideal system adjustment would be a software correction if a functional relationship could be identified which consistently reduced the radar lag. This functional relationship also had to reduce the radar lag to ± 15 MRAD, the ASCOT lock-on constant. Average radar lag was calculated by subtracting film position from radar position for each pass. It was noted that the average radar lag varies considerably between passes on the same mission.

If it is assumed that radar tracking error is a function of line-of-sight rate, the general form of the error is:

$$E_\varepsilon = A_1 + B_1\dot{\varepsilon}_R + C_1 q$$

$$E_\eta = A_2 + B_2\dot{\eta}_R + C_2 r$$

where
- E - Error
- $\dot{\varepsilon}_R$ - Target elevation rate of change with respect to interceptor airframe axis
- $\dot{\eta}_R$ - Target azimuth rate of change with respect to interceptor airframe axis
- q - Interceptor pitch angle rate
- r - Interceptor yaw angle rate
- A,B,C, - Constants derived by regression program

The target crossing and consequent radar tracking error was assumed to be a combination of a bias, A, plus a constant, B, times the azimuth or elevation tracking rate

TABLE 12

Tracking Rates (η_F, ε_F) vs Mean Radar Error (μ)

			AZIMUTH					ELEVATION						
MD	DATE	PASS#	TURN	TRACK RATE	YAW	AVERAGE RADAR ERROR	STAND. DEVIATION	TRACK RATE	PITCH	AVERAGE RADAR ERROR	STAND. DEVIATION	Lock-On	ε	η
	1975													
75017	17 Jan.	1	R L	-.233	-.003	-51.1	18.3	.049	.124	87.8	10.7	1	OK	OK
		2	L R	.120	.036	38.8	10.8	.209	.191	121	10.3	1		
		3	R L	-.150	-.044	-17.9	10.7	.144	.127	85.6	12.7	1		
D		4	L R	.114	.039	41.5	9.24	.137	.024	92.1	11.0	1		
		5	R L	-.150	-0.45	-26.2	10.7	.127	.159	81.8	9.36	1		
		6	L R	.066	.052	27.1	7.16	.240	.074	148	8.42	1		
		7	R L	-.151	-.048	-36.9	9.80	.273	.180	138	9.57	1		
		9	R L	-.200	-.043	-38.0	7.37	.240	.055	105	9.73	1		
75027	27 Jan.	2	R L	-.082	-.036	-3.0	11.1	.168	.054	28.5	15.6	1	OK	OK
		3	L R	.163	.054	55.5	14.1	.092	.138	31.1	12.5	1		
		5	R L	-.161	-.030	-18.8	12.2	.168	.026	51.3	18.0	1		
F		6	L R	.103	.054	43.8	15.0	.061	.118	36.2	15.3	1		
		7	R L	-.180	-.029	-27.1	12.5	.171	.097	56.8	12.7	1		
		8	L R	.097	.027	40.0	12.4	.020	.072	27.1	15.2	1		
		9	L R	.095	.056	48.4	16.0	.069	.075	35.1	16.6	1		
		10	R L	-.023	-.020	10.3	13.4	.027	.052	27.0	15.0	1		
95044	13 Feb.	3	R L	-.101	-.035	-53.7	8.92	.076	.042	-3.3	11.1	1	OK	OK
		4	L R	.085	.020	6.2	8.58	.031	.034	-11.0	10.7	1		
		5	L R	.111	.028	6.1	10.2	.042	.016	-3.8	9.50	1		
F		6	R L	-.099	-.027	-55.3	9.16	.057	.022	-10.7	8.80	1		
		8	R L	-.198	-0.18	-128	17.3	.081	.041	-6.4	8.95	1		
		9	R L	-.232	-.030	-152	10.9	.121	-.006	3.5	12.2	1		
75044	13 Feb.	2	L R	.078	.04	-9.2	8.29	.092	.016	-1.0	8.56	1	OK	OK
		3	R L	-0.35	-0.47	-52.1	9.59	.066	.048	-14.5	7.29	1	OK	OK
		5	R L	-0.59	-0.44	-54.7	7.97	.052	.082	-21.0	8.48	1	OK	OK
F		6	L R	.047	.046	-17.7	6.55	.088	-.001	-8.0	7.50	1	OK	OK
		7	L R	.053	.050	-15.9	6.68	.130	.045	2.9	9.54	1	OK	OK
		9	R L	.008	.070	-42.3	10.0	.045	.091	-17.8	7.87	1	OK	OK
		10	L R	.116	.044	49.4	26.0	.063	.081	22.0	18.0	1	?	?
75058	27 Feb.	1	R L	-.135	-.018	-79.8	14.6	.073	.006	-10.5	16.2	1	OK	OK
		2	L R.	.165	.024	1.2	14.0	.062	.02	-8.8	12.6	1		
F		4	L R	.162	.023	-5.7	13.3	.092	-.008	-5.9	15.2	1		
		6	R L	-.126	-.024	-79.0	15.0	.069	.070	-12.2	14.0	1		
		7	L R	.218	.023	-10.6	18.0	.049	.071	-12.8	17.1	1		
		9	R L	-.1[illegible]	-.022	-103	22.8	.088	.053	-14.7	23.4	1		
75336	2 Dec.	1	L R	.059	.044	7.97	3.15	.011	.078	4.93	4.12	1	2	OK
		2	R L	-.055	-.042	-20.3	3.99	.035	.083	11.6	2.91	1	OK	OK
F		3	L R	.076	.039	8.56	3.10	.022	.087	5.8	3.01	1	OK	OK
		4	R L	-.100	-.033	-22.5	2.91	.076	.048	10.13	2.54	1	OK	OK
75338	4 Dec.	1	R L	-.091	-.018	-32.0	4.48	.052	.116	9.5	2.62	1	3	3
		2	L R	.123	.053	12.3	3.89	.062	.065	8.8	2.75	1	OK	OK
		3	R L	-.063	-.020	-32.8	6.34	.063	.089	-5.8	6.21	1	OK	OK
		4	L R	.107	.047	17.9	26.6	.065	.067	16.5	24.5	1	OK	OK

* See explanation of numbers at end of Table.

TABLE 12

Tracking Rates (η_F, ε_F) vs Mean Radar Error (μ) (Continued)

			AZIMUTH					ELEVATION						
MD	DATE	PASS#	TURN	TRACK RATE	YAW	AVERAGE RADAR ERROR	STAND. DEVIATION	TRACK RATE	PITCH	AVERAGE RADAR ERROR	STAND. DEVIATION	Lock-On	ε	η
75345	11 Dec.	1	R L	-.032	-.008	-12.2	3.33	.018	.126	2.0	4.48		2,6	OK
		2	L R	.007	.012	1.4	4.28	-.011	.097	7.3	3.24		2,6	6
F		3	R L	-0.19	-0.17	-10.4	3.66	.036	.104	4.34	3.57		2,6	OK
		4	L R	-.004	.026	-4.7	3.63	-.008	.114	6.45	3.77		2,6	2
		5	R L	-.020	-.0005	-11.56	3.60	.020	.109	5.37	4.52		2,6	OK
75346	12 Dec.	1	L R	.050	-.009	5.2	7.64	-.025	.138	4.5	8.49		2,6	OK
		2	R L	-.022	-.008	-8.8	9.90	.013	.156	3.5	4.95		2	2,6
F		4	R L	-.029	-.002	-10.96	7.41	.018	.171	5.93	6.21		2,6	OK
		5	L R	.018	.023	-0.4	8.41	-.006	.156	4.02	8.49		2	6
		6	L R	-.026	.060	47.2	24.3	.006	.182	9.57	16.6		6	7or1
		7	L R	.008	.027	-4.42	15.0	.003	.157	3.98	17.2		6	6
75349-D	15 Dec.	1	L R	.020	.053	.06	3.59	.007	.105	5.41	2.30			OK
75353	19 Dec.	1	L R	.192	.025	48.4	5.04	.011	.055	6.92	7.74		6	OK
		2	R L	-.048	-.055	-4.1	6.94	.070	.071	22.3	4.64		OK	OK
F		3	L R	.106	.035	10.1	4.66	.036	.053	8.8	5.80		OK	OK
		4	R L	-.096	-.037	-24.1	3.62	.051	.070	19.2	4.65		OK	OK
		5	L R	.087	.045	16.85	3.69	.022	.106	16.76	5.00		OK	OK
		6	L R	.104	.033	20.9	3.27	.014	.072	9.74	4.62		OK	OK
		7	R L	-.193	-.045	-48.3	5.39	.081	.173	30.55	10.83		OK	OK
76022	1976 22 Jan.	1	L R	.156	.042	127.5	19.6	.131	.040	123	27.3		7or1	7or1
F		4	L R	.022	.030	-13.9	5.95	.033	.040	-13.4	7.67		OK	OK
76057	26 Feb.	2	L R	.006	.033	-.31	7.34	.010	.111	16.6	6.4		6	6
		3	L R	.052	.042	6.92	6.54	.036	.155	30.7	4.00		OK	OK
		4	L R	.058	.040	9.60	5.38	.067	.094	40.6	7.39		OK	OK
		5	R L	-.005	.041	-11.6	13.7	.074	.112	39.8	6.18		OK	6
		6	R L	-.016	.004	-20.6	30.6	-.005	.105	6.97	18.1		6,2	6
		7	R L	-.026	.028	-24.9	10.3	-.038	.182	1.67	12.1		6,2	6
76058	27 Feb.	1	R L	-.097	-.042	-41.3	3.97	.009	.150	13.0	6.24		OK	OK
		3	R L	-.044	-.054	-29.5	4.55	.030	.087	18.4	5.24		OK	OK
		4	L R	.080	.042	7.74	2.55	.122	.030	25.7	2.85		OK	OK
D		7	R L	-0.59	-.039	-33.1	4.48	.052	.113	16.2	6.93		OK	OK
		8	L R	.136	-.033	-5.82	47.3	.088	.076	-39.7	17.0		9	7or1
		9	R L	-.060	-.042	-91.5	27.5	.091	.071	79.9	40.1		7or1	7or1
76064	4 Mar.	1	L R	.046	.046			.017	.055				OK	OK
		2	R L	-.014	-.023			.041	.055				OK	2,6
F		4	R L	-.048	-.055			.104	.046				OK	OK
		5	L R	.102	.038			.069	.092				OK	OK
		6	R L	-.006	-.022			.040	.097				6	6
76065	5 Mar.	1	L R	.048	.042	-4.55	9.66	.028	.059	14.8	7.01		OK	OK
		2	R L	-.018	-.026	-24.3	4.24	.031	.073	8.45	5.12		OK	OK
		3	L R	.053	.036	-3.73	9.86	.069	.120	33.2	5.33		OK	OK
D		4	R L	-.072	-.055	-56.0	7.10	.047	.093	33.8	4.00		OK	OK
		6	R L	-.066	-.043	-36.3	4.26	.068	.081	19.2	5.38		OK	OK

TABLE 12

Tracking Rates (η_F, ε_F) vs Mean Radar Error (μ) (Continued)

			AZIMUTH					ELEVATION						
MD	DATE	PASS#	TURN	TRACK RATE	YAW	AVERAGE RADAR ERROR	STAND. DEVIATION	TRACK RATE	PITCH	AVERAGE RADAR ERROR	STAND. DEVIATION	Lock-On	ε	η
76065	5 Mar.	7	L R	.162	.042	104	26.5	.159	.222	141	28.4	1	7	7
		8	R L	-.116	-.045	-53.3	3.96	.127	.080	34.1	5.46	1	OK	OK
76070	10 Mar.	2	R L	-.003	0.17	-2.28	13.5	-.030	.139	-8.68	4.69	1	OK	6,2
		3	L R	.091	-.052	12.2	14.2	.084	.154	7.77	4.69	1	OK	2
D		4	R L	0.081	-.024	-9.29	20.2	.050	.097	-.13	5.05	1	OK	8
		5	L R	.101	.057	.01	14.6	.149	.029	11.9	5.26	1	OK	8
		6	R L	-.164	-.017	-1.85	28.5	.121	.075	11.3	5.83	1	OK	8
76077	17 Mar.	1	L R	.035	.035	3.54	3.40	.041	.074	9.8	4.41	1	OK	OK
		2	R L	-.030	-.034	-10.5	3.36	.065	.047	10.5	3.58	1	OK	OK
		3	R L	-.036	-.041	-12.7	6.74	.086	.061	16.4	5.69	1	OK	OK
D		5	L R	.118	.037	3.0	4.05	.086	.093	6.4	7.12	1	OK	OK
		6	R L	-.131	-.024	-27.4	3.28	.101	.105	12.7	3.93	1	OK	OK
		7	L R	.164	.037	9.8	4.09	.144	.106	13.1	5.79	1	OK	OK
76078	18 Mar.	1	R L	-.024	-.017	-7.1	3.02	.034	.066	3.4	3.47	1	OK	OK
		2	L R	.073	.017	16.6	4.55	.065	.105	16.6	4.90	1	OK	OK
		3	L R	.006	.051	5.9	6.14	.007	.101	13.7	9.59	1	OK	OK
D		4	L R	.025	.024	10.5	3.55	.017	.094	13.4	3.84	1	OK	OK
		5	R L	-.046	-.013	-16.8	11.4	.026	.087	-6.4	9.59	1	OK	OK
		6	L R	.170	.025	35.9	6.75	.122	.101	30.3	7.36	1	OK	OK
		7	R L	-.139	-.029	-33.4	3.67	.118	.079	11.4	7.95	1	OK	OK
		8	R L	-.169	-.021	-41.8	3.58	.101	.056	17.6	2.96	1	OK	OK
76083	23 Mar.	1	L R	.168	.040	26.2	4.42	.105	.087	27.0	3.19	1	OK	OK
		2	R L	-.162	-.022	-29.7	7.89	.103	.034	2.9	7.47	1	OK	OK
F		3	L R	.179	.036	24.4	2.59	.130	.065	23.3	2.51	1	OK	OK
		4	R L	-.116	-.031	-35.0	9.16	.114	.049	15.7	6.35	1	OK	OK
		5	L R	.303	.032	58.6	38.3	.257	.069	59.1	28.9	1	9	9
76085-F	25 Mar.	2	L R	.016	.055	4.4	7.74	.020	[illegible]	25.2	5.82	1	OK	OK
76092	1 Apr.	1	R L	-.033	-.0007	-26.8	16.4	-.005	.102	20.0	7.54	1	2	OK
F		3	R L	-.005	.046	4.3	1.91	-.053	.120	12.3	3.76	1	OK	OK
79093	2 Apr.	1	R L	-.030	.012	-17.1	10.7	-.004	.121	9.8	14.1	1	6	OK
		2	L R	.002	-.038	-32.0	4.03	-.080	.207	56.1	4.42	1	OK	OK
D		3	L R	.005	.039	167.5	3.11	-.055	.135	-83.4	28.6	1	10	10
		5	R L	.0003	-.041	211.1	3.82	-.030	.099	-90.6	21.5	1	10	10
76110	19 Apr	1				-30.2	32.8			-79.2	20.9		1	1
		2	R L			23.3	30.2			-108	24.0		1	1
F		3	L R			-28.3	30.5			-97.3	32.1		1	1
		4	L R			-30.9	22.9			-78.9	35.1		1	1
		5	R L			33.3	41.6			-113	39.8		1	1
		6	L R			-22.0	14.5			-77.9	38.2		1	1
		7	R L			32.4	42.2			-120	26.7		1	1
		8	R L			22.2	38.9			-103	22.7		1	1

TABLE 12

Tracking Rates (η_F, ϵ_F) vs Mean Radar Error (μ) (Continued)

			AZIMUTH					ELEVATION						
MD	DATE	PASS#	TURN	TRACK RATE	YAW	AVERAGE RADAR ERROR	STAND. DEVIATION	TRACK RATE	PITCH	AVERAGE RADAR ERROR	STAND. DEVIATION	Lock-On	ϵ	η
76111	20 Apr.	1	R L	-.090	-.033	-18.9	4.14	.103	.012	12.6	5.21	1	OK	OK
		2	L R	.059	.033	19.2	2.50	.064	.026	10.6	2.86	1	OK	OK
F		3	R L	-.181	-.029	-38.8	4.36	.077	.076	11.9	3.84	1	OK	OK
		4	R L	-.151	-.038	-28.4	4.75	.134	.069	22.5	6.49	1	OK	OK
		5	L R	.180	.027	40.9	4.53	.092	.117	20.4	4.62	1	OK	OK
76118	27 Apr.	1	R L	-.085	-.031	-23.1	2.64	.081	.019	16.9	4.75	1	OK	OK
		2	L R	.074	.054	20.4	7.18	.130	.0005	21.8	6.15	1	OK	OK
		3	R L	-.123	-.032	-33.1	3.57	.104	.055	25.4	6.76	1	OK	OK
F		4	L R	.129	.030	13.9	3.24	.105	.035	6.6	5.84	1	OK	OK
		5	L R	.110	.043	27.7	2.89	.110	.007	19.1	4.03	1	OK	OK
		6	R L	-.164	-.036	-52	9.23	.182	.040	51.7	16.3	0	2	OK
		7	L R	.186	.042	-57	3.23	.172	.015	34.2	6.47	0	2	OK
		8	R L	-.132	-.037	-75	23.3	.128	.118	130	58.0	0	1,2	1
76119	28 Apr.	4	L R	.044	.016	-229	53.6	.007	.070	-295	56.5	1	7	7
		5	L R	.037	.037	9.0	3.13	.019	.042	3.3	3.97	1	6	OK
		6	L R	.052	.052	7.72	3.28	.064	.034	13.8	5.58	1	OK	OK
F		7	R L	-.034	-.040	-12.9	4.22	.077	.055	22.2	5.60	1	OK	OK
		8	L R	.042	.033	16.8	12.4	-.004	.084	9.7	11.4	1	2	OK
		9	L R	.093	.039	27.9	5.53	.021	.085	23.0	6.42	1	6,2	OK
		10	L R	.103	.042	35.1	5.37	.105	.043	29.1	8.14	1	OK	OK
76121	30 Apr.	1	L R	-.003	.035	4.9	4.28	.033	.054	2.2	3.61	1	OK	OK
		2	R L	-.022	-.022	-11.6	3.79	.035	.048	.3	3.51	1	OK	OK
		3	L R	.025	.039	9.9	2.83	.022	.058	2.9	2.59	1	OK	OK
F		4	L R	.031	.043	10.3	6.20	.050	.091	6.4	11.9	1	OK	OK
		5	L R	.135	.040	42	3.88	.115	.012	27.6	4.60	1	OK	OK
		6	R L	-.098	-.035	-24	3.31	.159	.036	22	3.39	0	OK	OK
		7	L R	.082	.042	-18.8	5.11	.088	.065	15.5	6.29	1	OK	OK
		8	R L	-.090	-.036	-17.7	4.27	.106	.012	-.6	4.06	1	OK	OK
		9	L R	.140	.047	39.5	4.46	.161	.042	37.6	6.80	1	OK	OK
76126	5 May	1	L R	.034	.036	11.3	2.82	.033	.084	4.9	4.21	1	2	OK
		3	R L	-.062	-.029	-23.7	2.90	.059	.020	-2.6	3.36	1	OK	OK
D		4	L R	.060	.030	11.7	3.73	.045	.039	4.7	3.75	1	OK	OK
76131	10 May	1	L R	.032	.043	15.4	20.5	.055	.067	11.4	11.6	1	OK	OK
		2	R L	-.030	0.028	-14.5	2.65	.059	.032	-3.5	3.30	1	OK	OK
		3	L R	.117	.050	24.8	5.37	.102	.138	39.5	17.7	0	7	OK
		4	R L	-.057	-.026	-21.1	5.22	.060	.105	10.8	4.17	1	OK	OK
		5	R L	-.115	-.035	-22.7	2.30	.117	.085	17.1	5.98	1	OK	OK
		6	L R	.039	.049	12.0	3.75	.045	.073	5.2	3.92	1	OK	OK
		7	R L	-.143	-.035	-49	5.47	.121	.093	33.9	9.04	1	OK	OK
		8		.104	.037	23.9	5.06	.182	.103	66.1	11.5	0	7	OK
		9	L R	.007	.029	-4.6	5.85	.020	.090	-3.1	3.25	1	2	OK
		10	R L	-.064	-.027	-18.1	4.39	.148	.104	9.1	5.82	1	OK	OK

TABLE 12

Tracking Rates (η_F, ε_F) vs Mean Radar Error (μ) (Continued)

			AZIMUTH						ELEVATION						
MD	DATE	PASS#	TURN		TRACK RATE	YAW	AVERAGE RADAR ERROR	STAND. DEVIATION	TRACK RATE	PITCH	AVERAGE RADAR ERROR	STAND. DEVIATION	Lock-On	ε	η
76139	18 May	1	R	L	.105	-.033	-299	54.8	.087	.054	150	27.1	0	/	/
		2	L	R	-.160	.049	227	25.2	.173	.147	121	27.1	0	/	/
		3	R	L	.241	-.033	-333	37.0	.125	.066	152	21.8	0	/	/
D		4	L	R	-.141	.054	267	37.7	.087	.108	176	22.6	0	/	/
		5	R	L	.410	-.031	-340	22.8	.401	.197	177	23.7	0	/	/
		6			-.218	.070	55.1	38.3	.284	.170	155	39.8	0	/	/
		7	L	R	.008	.053	12.9	17.9	.057	.082	73.8	81.3	0	/	6
76162	10 Jun.	1	R	L	-.065	-.078	31.2	126	.086	.013	39.2	53.3		OK	8
		2	R	L	-.068	-.017	-32.5	10.3	.125	.139	28.5	15.7		OK	OK
		3	R	L	-.072	-.033	-26.3	7.13	.175	.059	30.0	6.72		OK	OK
		4	R	L	-.015	-.049	-7.2	46.2	.119	.090	28.7	28.4		OK	OK
		5	R	L	-.073	-.027	-34.6	2.66	.250	.056	55.3	11.8		OK	OK
		6	R	L	-.121	-.017	-50.3	8.96	.129	.190	33.9	14.2		OK	OK
		7	R	L	-.101	-.053	6.7	100	.110	.076	24.2	25.5		OK	8
76162	10 Jun.	11	R	L	-.065	-.048	-22.7	18.5	.087	.068	25.6	22.2		OK	OK
		12	R	L	-.070	-.026	-25.2	12.1	.127	.129	36.3	15.4		OK	OK
		13	R	L	-.072	-.043	-19.5	8.01	.176	.049	37.7	8.02		OK	OK
F		14	R	L	-.017	-.050	-6.5	11.0	.119	.093	32.2	16.9		OK	OK
		15	R	L	-.073	-.032	-27.5	7.14	.246	.045	60.3	17.8		OK	OK
		16	R	L	-.121	-.025	-41.1	10.0	.130	.183	40.2	14.6		OK	OK
		17	R	L	-.103	-.031	24.1	10.2	.110	.112	32.1	16.6		OK	OK
76163	11 Jun.	1	L	R	.055	.031	29.6	5.93	.057	.042	18.3	7.08		OK	OK
		2													
		3	R	L	-.131	-.034	-27.1	5.56	.117	.129	23.4	6.75		OK	OK
		4	L	R	.115	.030	41.9	7.36	.104	.026	25.7	8.69		OK	OK
		5	L	R	.063	.034	32.0	5.25	.090	.019	28.2	8.52		OK	OK
F		6	R	L	-.163	-.043	-32.7	7.96	.188	.028	38.5	11.4		OK	OK
		7	R	L	-.040	-.051	2.1	6.52	.051	.045	13.2	5.29		OK	OK
		8	L	R	.136	.025	44.6	6.49	.092	.034	26.8	6.18		OK	OK
		9	L	R	.094	.038	37.6	4.88	.130	.006	30.3	8.72		OK	OK
76166	14 Jun.	1	R	L	-.034	-.016	-12.0	6.18	.009	.079	-5.1	6.84		12	12
		2	L	R	.004	.045	2.0	2.01	.023	.059	2.2	7.22		12	12
		3	R	L	-.050	-.052	-18.6	8.35	.115	.020	11.6	6.81		OK	OK
F		4	L	R	.049	.015	27.1		.056	.101	17.8	37.0		OK	OK
		5	R	L	-.074	-.038	-26.1	46.8	.060	.087	2.3	5.70		OK	OK
		6	L	R	.182	.042	23.7	7.51	.060	.151	6.0	6.98		OK	OK
		7	L	R	.096	.048	12.6	5.97	.086	.090	7.3	6.49		OK	OK
		8	L	R	.100	.031	23.0	5.71	.134	.113	21.6	5.05		OK	OK
76167	15 Jun.	1	L	R	.181	.032	-81	29.5	.087	.062	-87	20.6		OK	OK
D		2	L	R	-.013	.028	-8.2	5.17	-.071	.092	-33.2	27.6		OK	OK

TABLE 12

Tracking Rates (η_F, ε_F) vs Mean Radar Error (μ) (Continued)

			AZIMUTH					ELEVATION						
MD	DATE	PASS#	TURN	TRACK RATE	YAW	AVERAGE RADAR ERROR	STAND. DEVIATION	TRACK RATE	PITCH	AVERAGE RADAR ERROR	STAND. DEVIATION	Lock-On	ε	η
76174	22 Jun.	1	R L	-.041	-.006	-14.6	6.47	.039	.053	-6.5	5.77		OK	OK
		2	L R	.248	.016	65.8	11.4	.199	.041	48.5	11.3		OK	OK
D		3		-.000005	.036	129	296	.191	.054	-41.1	29.8		OK	11
		4	R L	-.149	.60.3	50.6	28.7	.171	.146	23.1	9.66		OK	OK
		5	R L	-.133	-.033	-52.1	6.68	.396	.027	52.9	8.71		OK	OK
76175	23 Jun.	1	R L	-.034	-.069	19.9	95.1	.082	-.00003	25.4	78.9		2,8	2,8
		2	R L	-.804	-.041	-42.1	9.11	.135	.107	13.1	13.9		OK	OK
F		3	R L	-.033	-.033	-21.9	6.44	.128	.028	8.3	5.24		OK	2
		4	R L	-.116	-.034	-46.3	4.87	.208	.063	26.9	14.2		2	OK
		5	R L	-.025	-.074	32.5	104	.079	-.020	40.0	76.8		8	8
		6	L R	.097	.042	23.4	6.38	.133	.016	14.5	7.28		OK	OK
		7	R L	-.121	-.029	-53.5	3.27	.176	.028	20.0	3.64		OK	OK
		8	L R	.235	.052	42.8	4.12	.064.	.142	14.5	2.32		OK	OK
		9	L R			47.5	5.85			13.8	4.18		OK	OK
76197	15 Jul.	1	L R	.083	.040	7.1	2.52	.081	.055	4.8	4.12		OK	OK
		2	R L	-.049	-.057	-14.1	55.8	.093	.056	11.9	48.1		2,8	2,8
		3	R L	-.064	-.104	38.6	125	.188	-.041	56.5	67.5		2,8	2,8
		4		.048	-.250	194	13.0	.086	-.284	134	24.3		2,8	2,8
F		5	R L	-.051	-.036	-32.0	3.90	.099	.031	1.5	6.06		OK	OK
		6	L R	.082	.045	7.4	3.56	.094	.030	5.8	3.28		OK	OK
		7	L R	.169	.034	23.0	4.95	.111	.066	14.1	3.46		OK	OK
		8		.133	.032	-22.8	30.9	.126	.135	15.1	4.57		OK	OK
		9		-.128	-.026			.145	.111				OK	OK

1. No radar lock-on - or lock-on lost.
2. Poly curve fit to radar data was not adequate for calculation of crossing rates.
3. No radar data at all.
4. Tape not read due to parity errors.
5. Suspect tape (data out of chronological order.
6. AZ rate or EL rate changed sign.
7. Radar not tracking properly (1)
8. Erratic radar data.
9. Film data too short (6 pts or less)
10. No radar motion
11. No film data
12. Radar data appears descretized.

of the radar antenna, plus a constant, C, times the pitch or yaw angle rates of the interceptor. The missions were divided into seven chronological groupings and a regression analysis was performed on each group to determine the coefficients A, B, and C. The data groupings (as defined in Section 3.3.3.1) are shown in Figures 21 thru 27.

The results of the regression indicated that the significant contribution was always the $B_1\dot{\varepsilon}_R$ term. This is not surprising as the pitch and yaw rates were small compared with $\dot{\varepsilon}$ as was the C coefficient. The total contribution of Cq or Cr was thus negligible. The A coefficient defines the bias in the radar. The bias contribution while small, varied between data groups indicating a negligible constant bias. Ideally if the radar lag is a true function of crossing rate, a zero crossing rate implies no radar lag. For these reasons the functional relationship for all missions was simply expressed as

$$E_\varepsilon = B_1\dot{\varepsilon}_R \quad ; \quad B_1 = -135$$

$$E_\eta = B_2\dot{\varepsilon}_R \quad ; \quad B_2 = -250$$

The mean radar error components were then calculated for each pass. The tracking error correction was applied and the mean radar error was again calculated. Histograms of the mean radar errors are shown in Figures 28 thru 31. The mean error plotted can be expressed as

$$\varepsilon_R - \varepsilon_F - E_\varepsilon = E_{\varepsilon c}$$

$$\eta_R - \eta_F - E_q = E_{\eta c}$$

where ε_R Radar Elevation

ε_F Film Elevation

η_R Radar Azimuth

E_ε Elevation Tracking Corrector

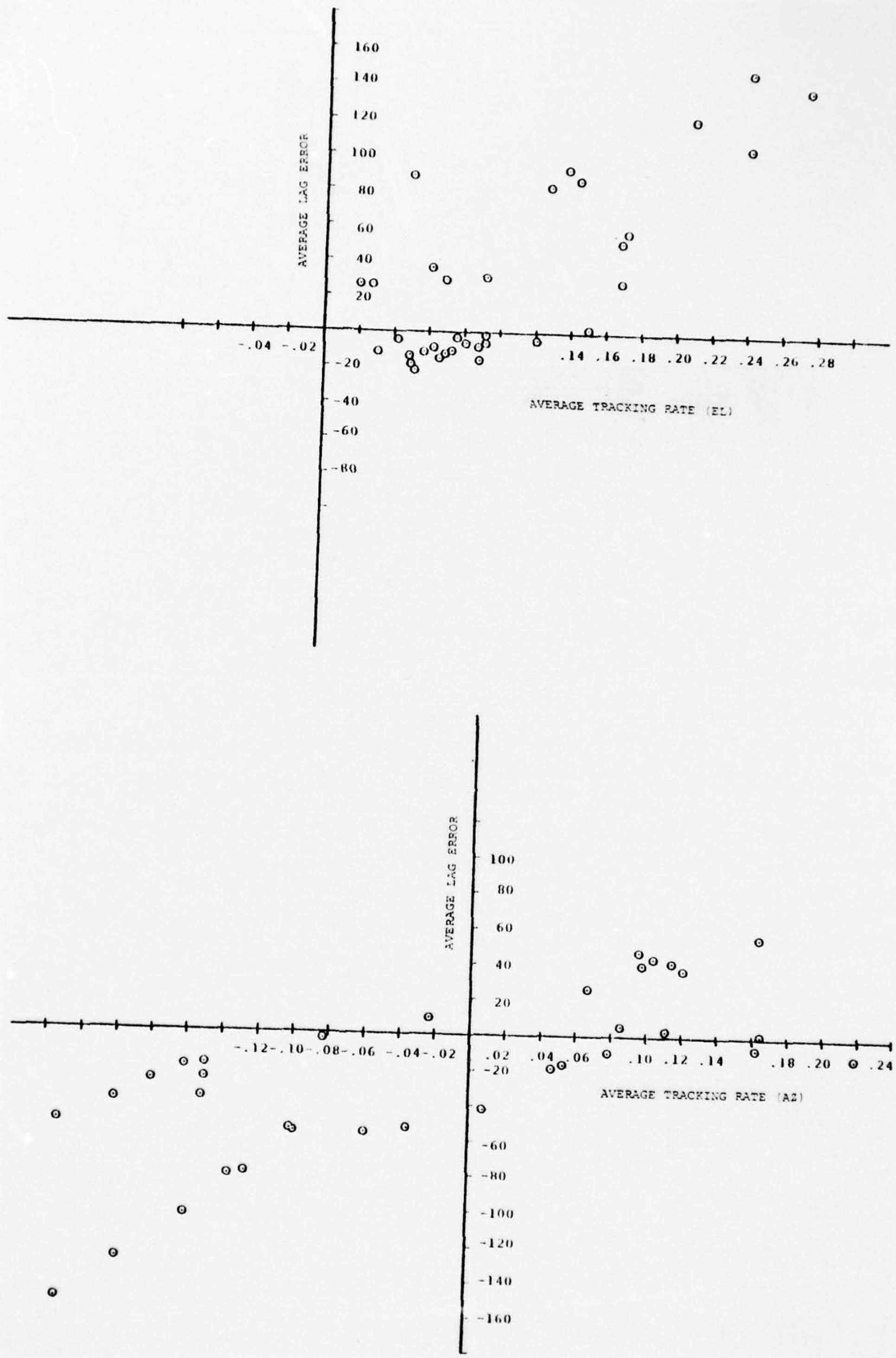
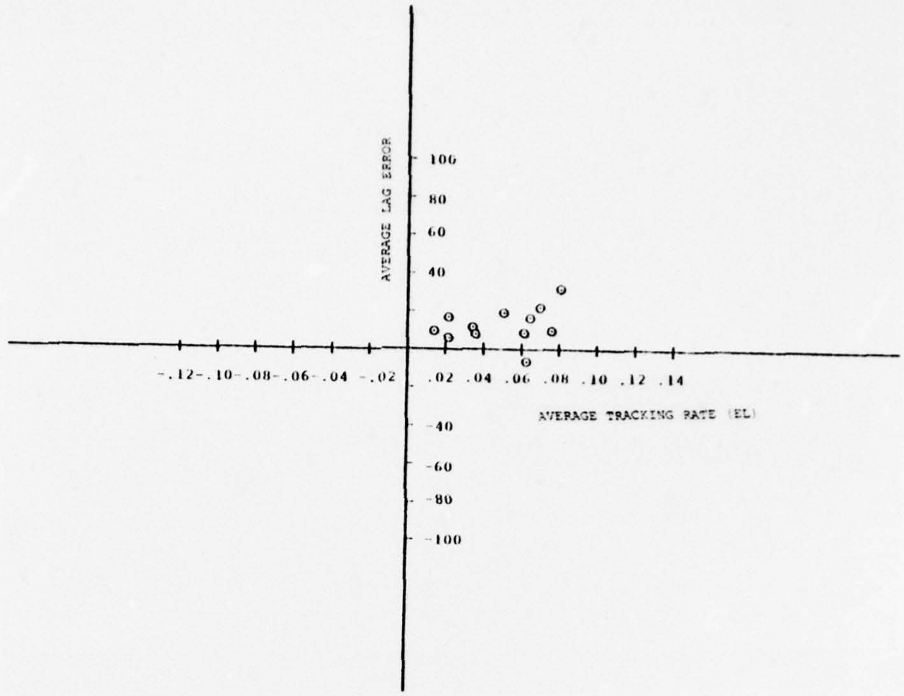


Figure 21 Data Group I



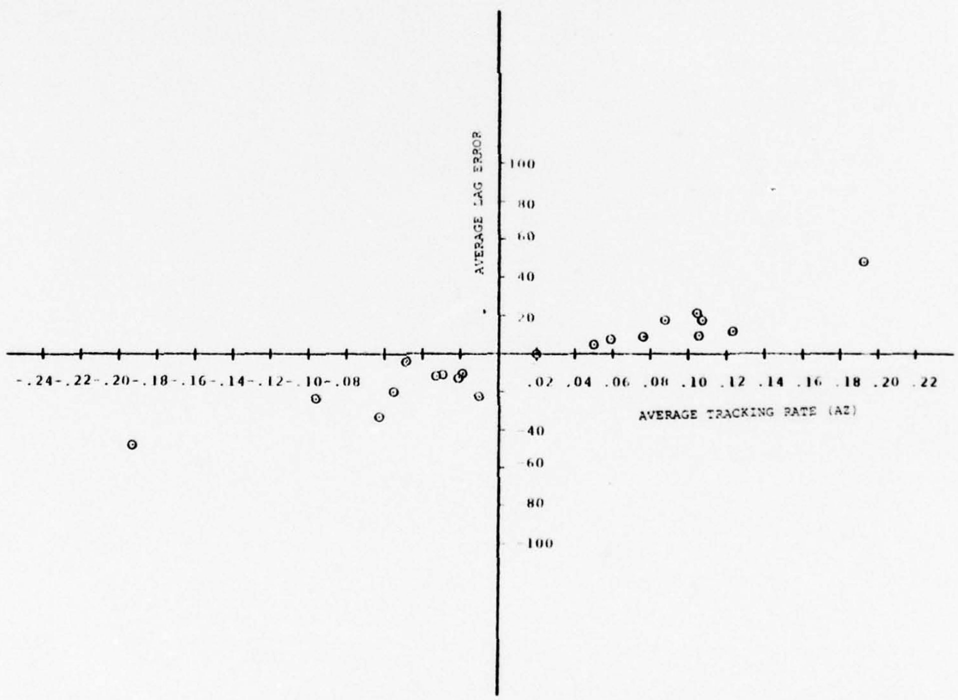
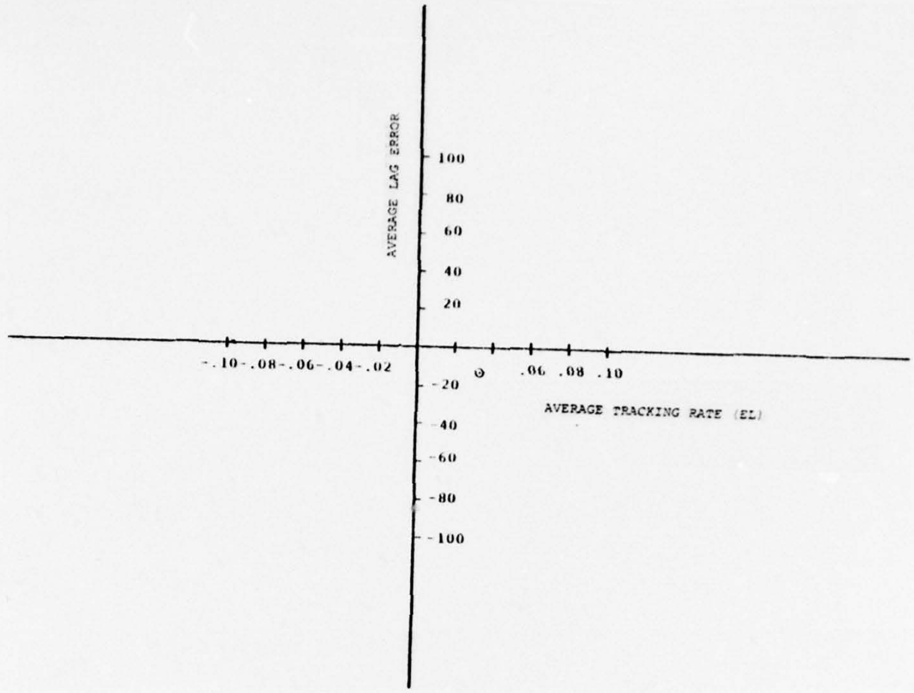


Figure 22 Data Group II



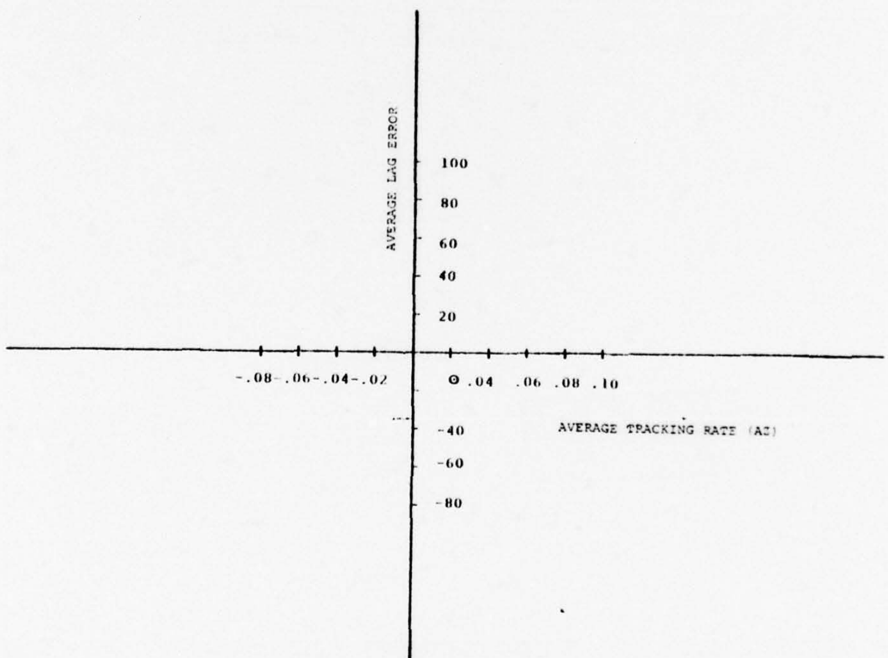
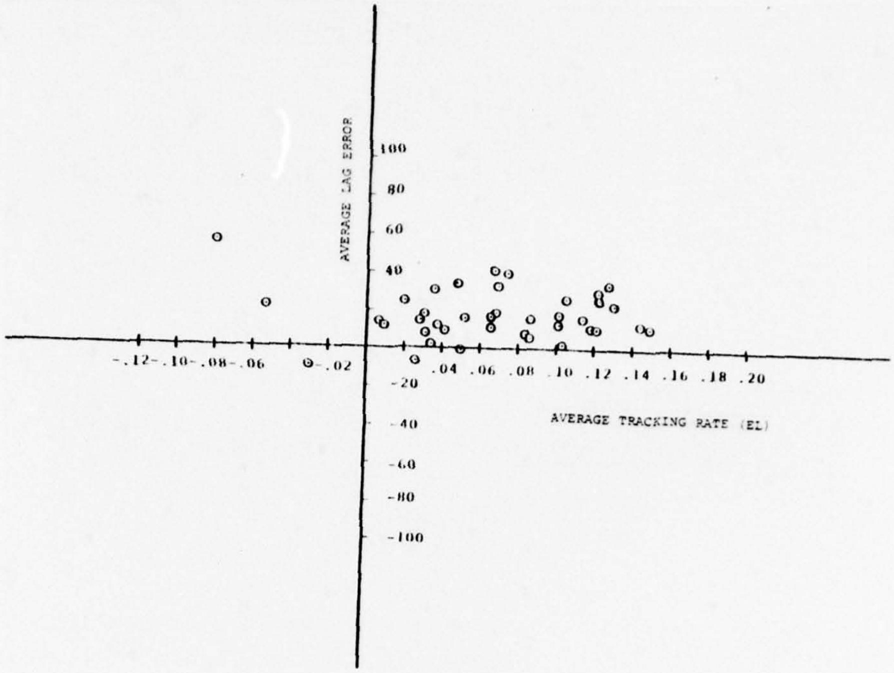


Figure 23 Data Group III



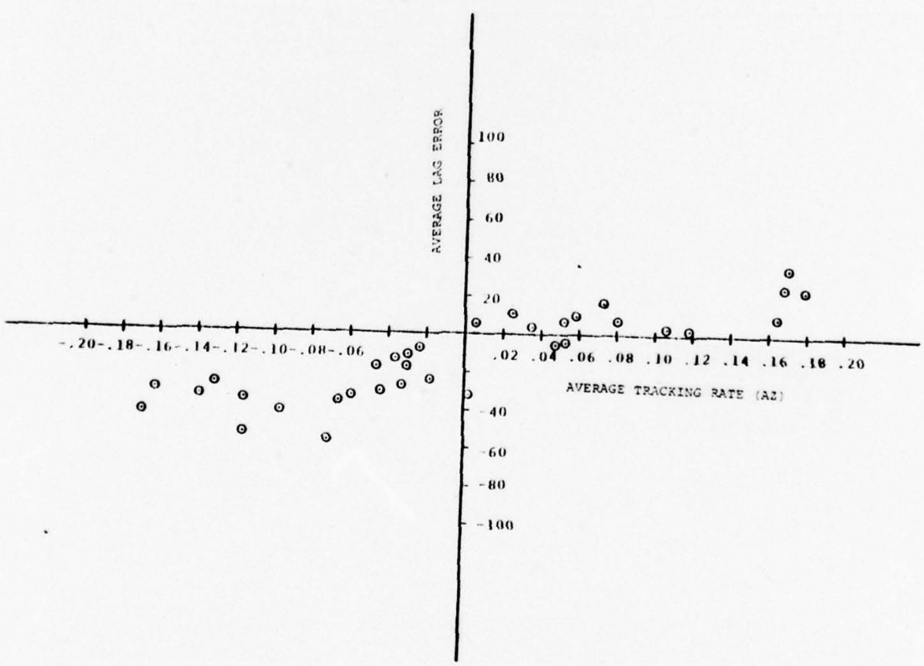
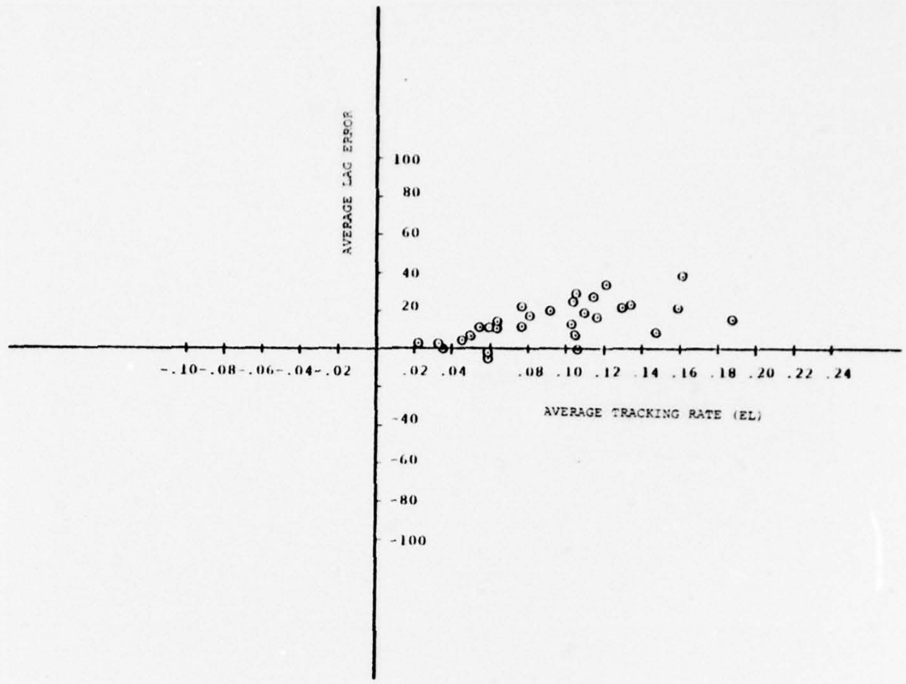


Figure 24 Data Group IV



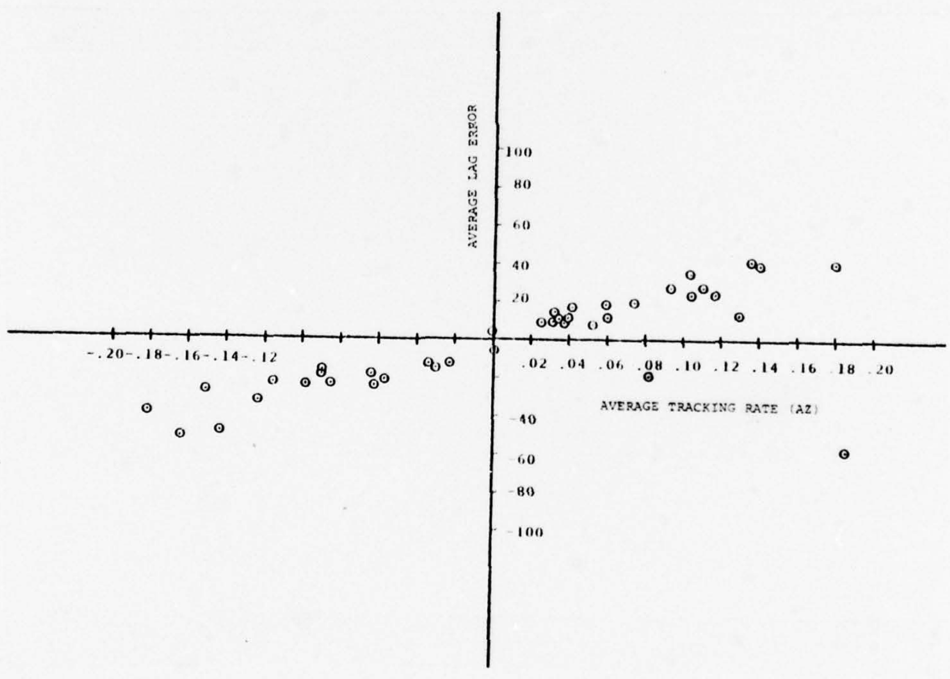
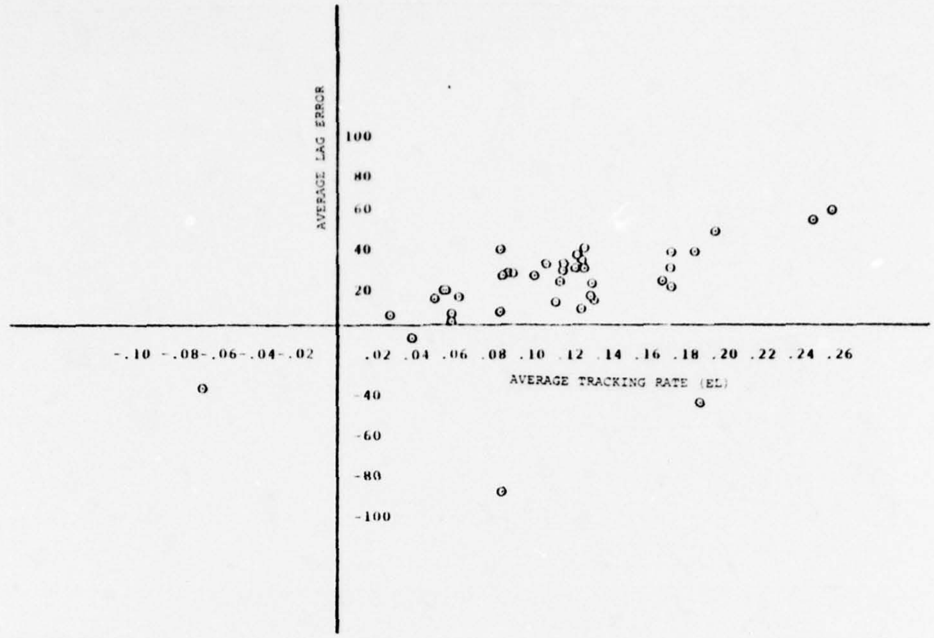


Figure 25 Data Group V



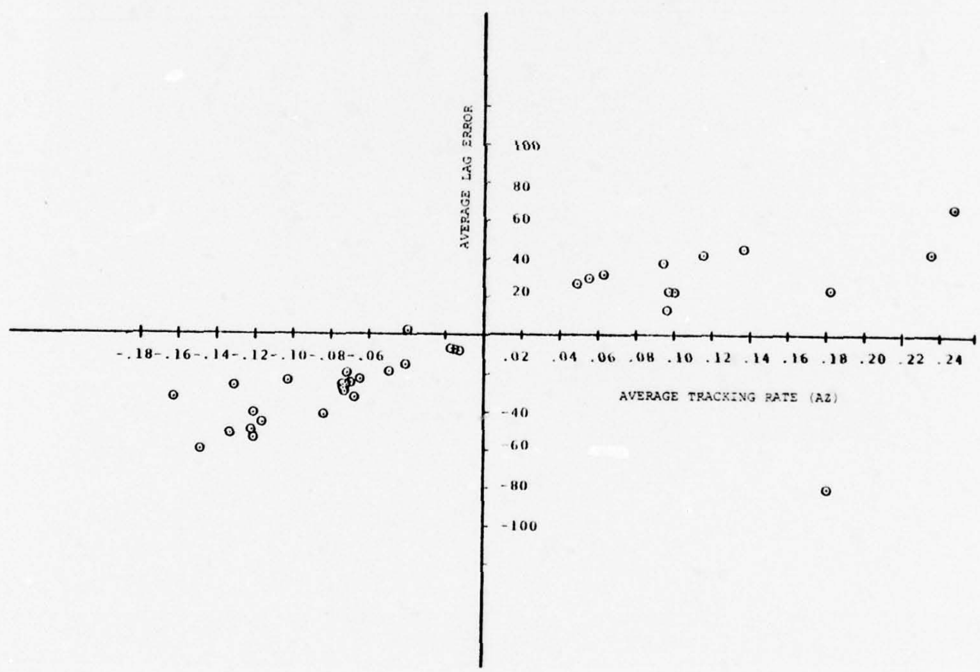
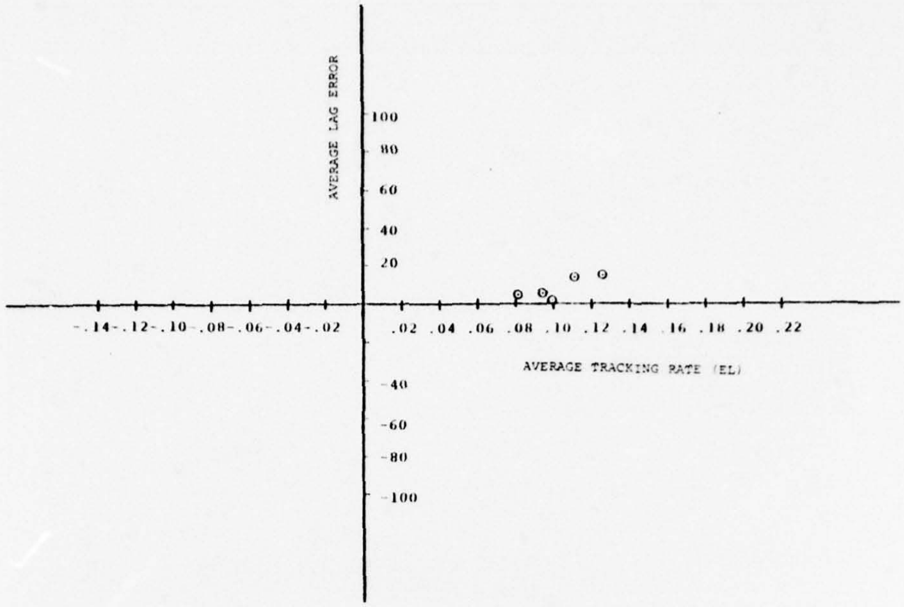


Figure 26 Data Group VI



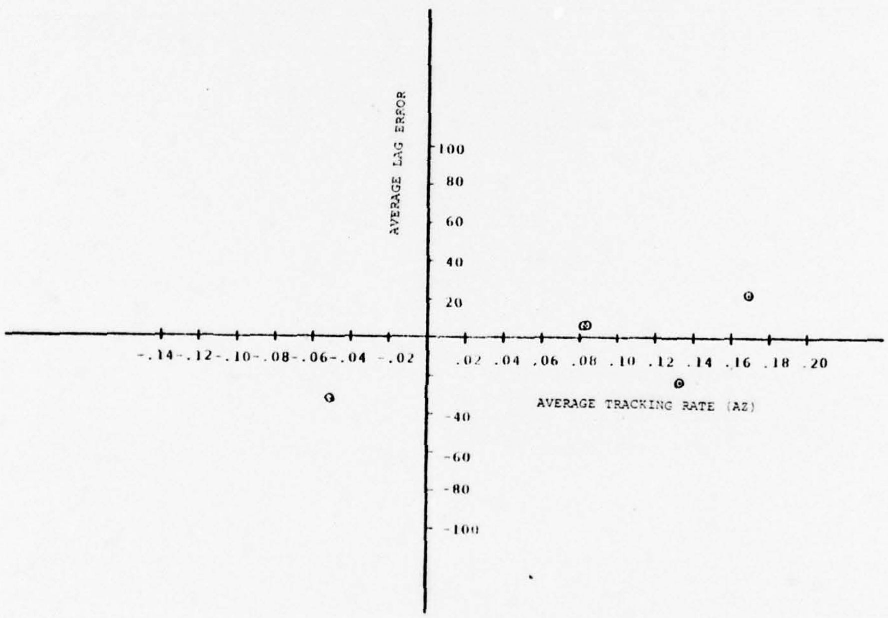


Figure 27 Data Group VII

Figure 28 Average EL Error Before Tracking Correction

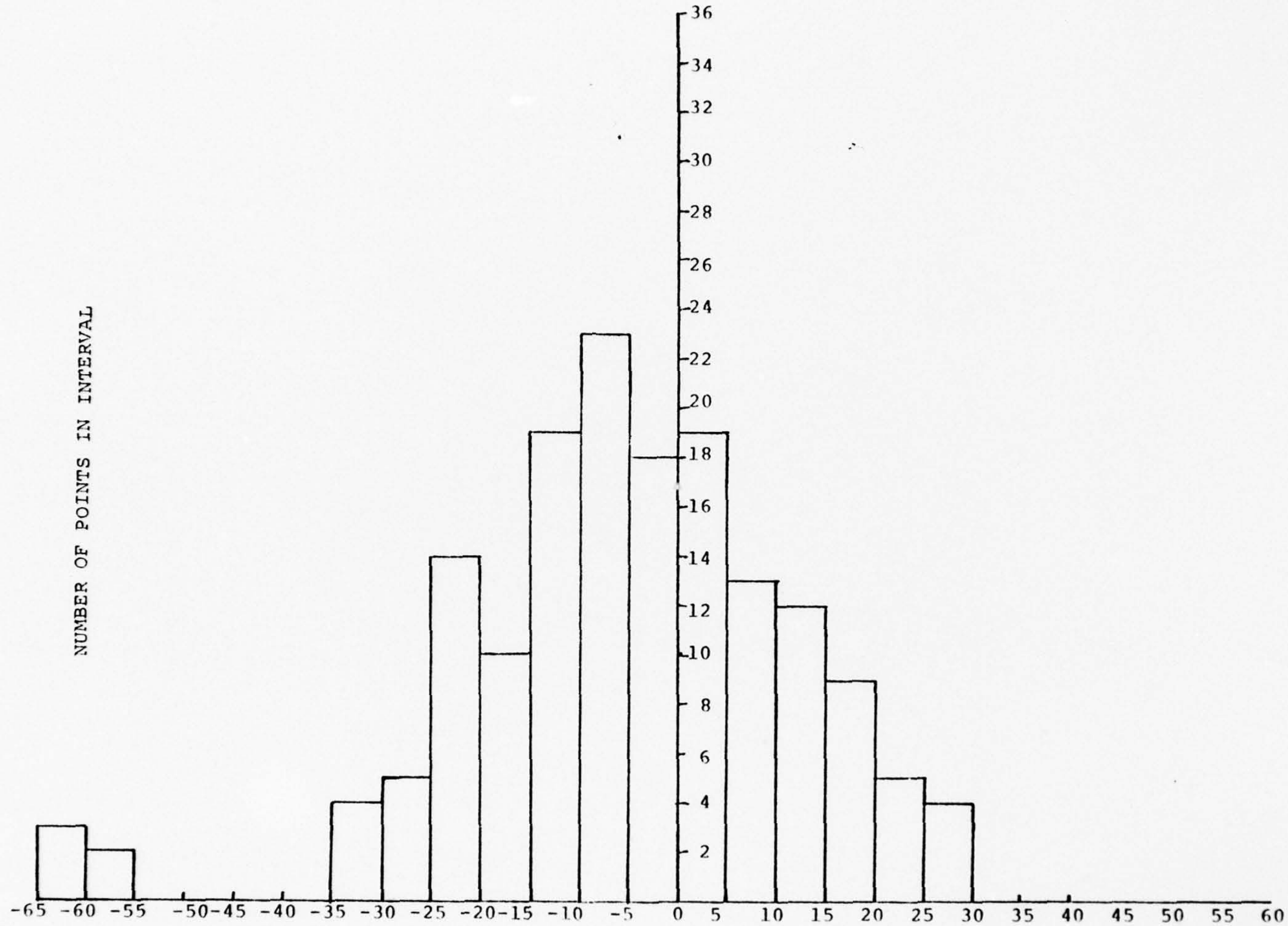


Figure 29 Average EL Error After Tracking Correction

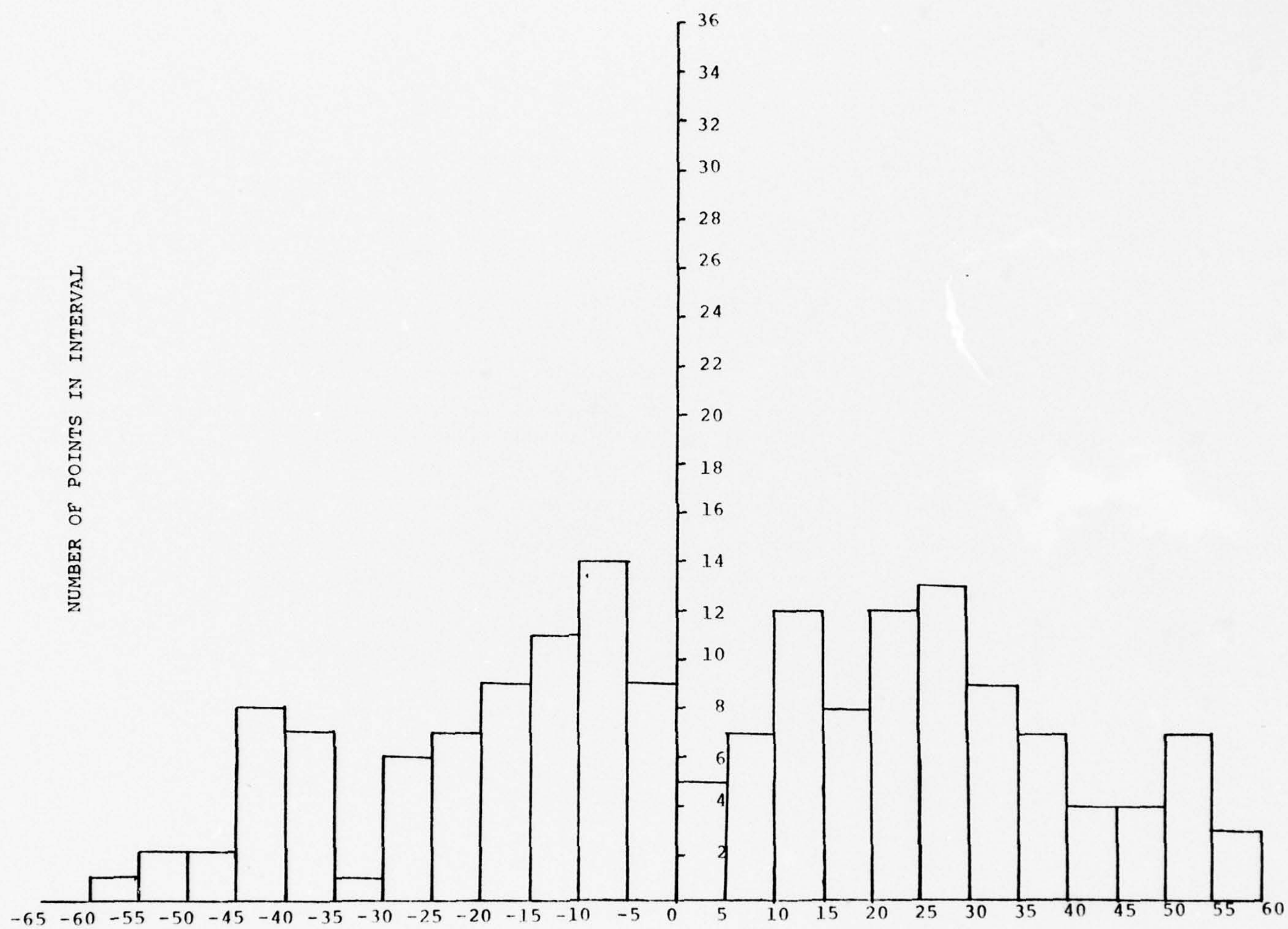


Figure 30 Average AZ Error Before Tracking Correction

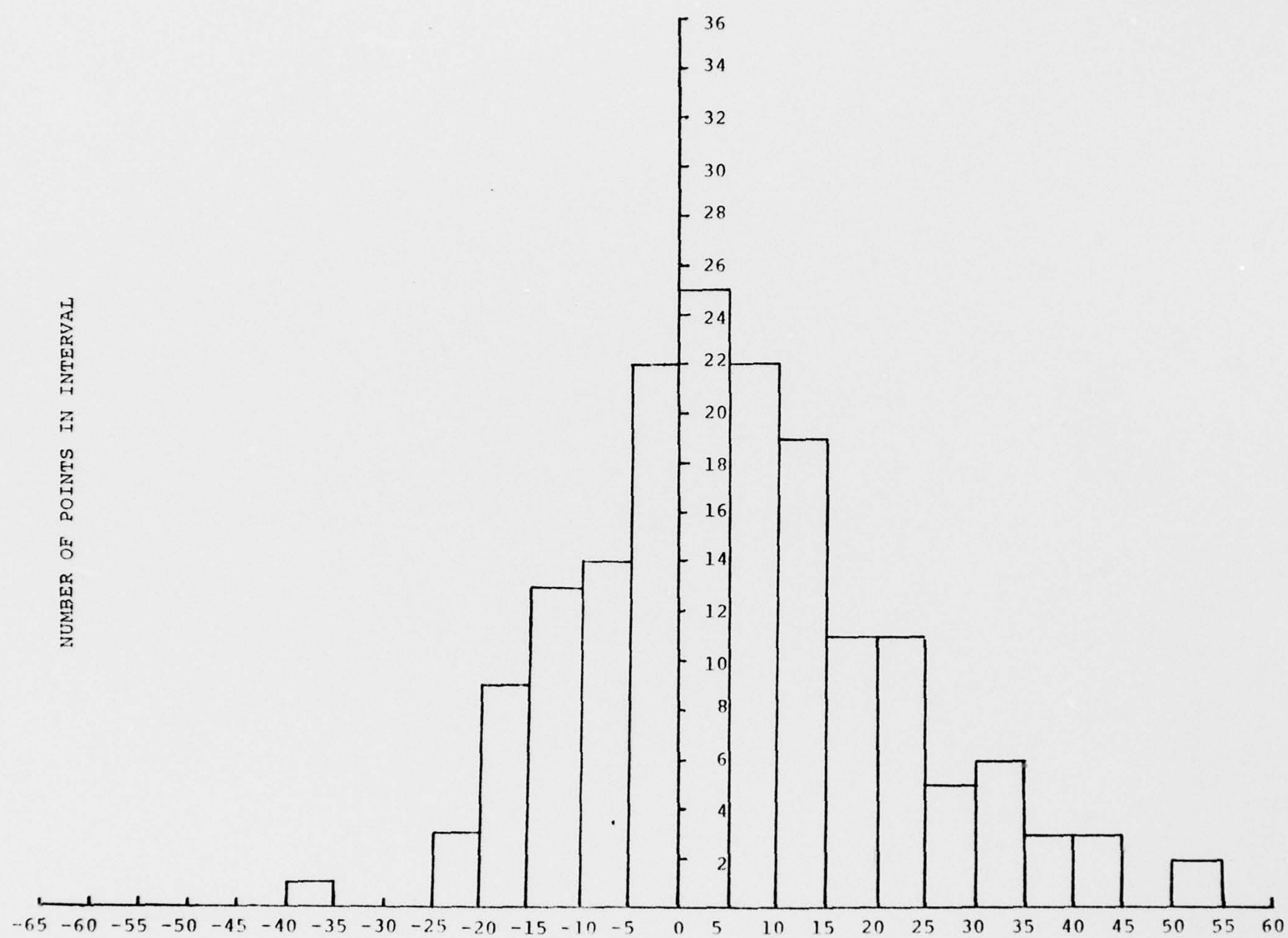


Figure 31 Average AZ Error After Tracking Correction

E_{η} Azimuth tracking Correction

E_{ϵ_c} Elevation error

E_{η_c} Azimuth error

The tracking error correction did reduce the radar mean lag for a number of the passes. However, a significant number of the passes were still greater than the 17 MRAD constraint. This suggests that error sources other than crossing rates are contributing to the radar lag.

3.3.4.2 Line-of-Sight Rate of Change Analysis

Validity of Sight Eval Data

To provide an alternative to graphical methods of studying the radar lag problem, a program, SDATA, was written to reduce the Sight Eval data. SDATA first smooths the Sight Eval data and then uses an Euler parameter integration scheme to compute the principal variables during each engagement. The important variables computed are range, range rate, pursuer and target positions and velocities, the absolute LOS turn rate and the LOS turn rate relative to the pursuer, pursuer turn rate, headings of both pursuer and target.

The program SDATA had a second purpose which was to determine if the Sight Eval data was invalid. Once the SDATA integration scheme is initialized, it is possible to compare the computed values of pursuer roll, pitch, and yaw angles and pursuer speed with those values obtained from smoothed Sight Eval data. These comparisons indicate that the Sight Eval data, although much noisier than would be hoped, does furnish a reasonable basis for a study of the radar lag problem, assuming, of course, that none of the scaling or other operations, performed on the tapes prior to our effort, introduced significant error. Documentation has not been obtained on some aspects of these operations. Two other qualifications are in order. The geometry of most of the engagements or passes in the Sight Eval data are very similar in geometry. This raises the question of the confidence one could have in a derived fit for a fundamentally different engagement geometry. Another problem is that most passes are of short duration, often less than a second, rarely over 2 seconds. Would a fit be valid for larger periods of time?

It was hoped that an analysis of a large number of engagements would allow the derivation of an equation giving radar lag angle as some function of the various engagement turn rates. Three turn rates that seem likely candidates are true

LOS turn rate with respect to inertial coordinates, pursuer ownship turn rate, and the relative LOS turn rate with respect to the pursuer.

With this goal in mind, a number of fitting equations were derived and added to SDATA. A large number of engagements were then analyzed in SDATA and the results of each of the fits for each engagement was tabulated. Two different types of fit were utilized and their deviation and intent will now be outlined.

In the following discussion all turn rates and the vector radar lag angle itself are expressed in terms of their components in the SDATA inertial x y z coordinate system.

Radar lag angle is represented as a vector as follows:

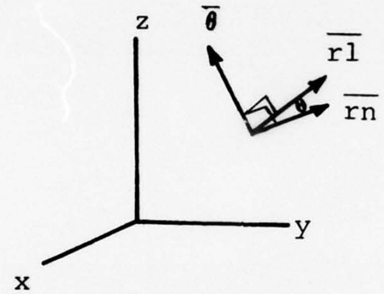


Figure 32

In Figure 32 $\overline{rn}$ and $\overline{rl}$ are unit vectors along the radar axis and along the true line of sight to the target respectively, both vectors emanating from the pursuing aircraft. Then by definition $\overline{\theta} = \theta \left|\dfrac{\overline{rn} \times \overline{rl}}{\overline{rn} \times \overline{rl}}\right|$. This procedure is valid since θ is of relatively small magnitude and it has the advantages of eliminating many annoying sign convention problems as well as simplifying the formulation of fit equations.

Let $\overline{wa}$ be the true LOS turn rate, $\overline{wb}$ the pursuer ownship turn rate, and $\overline{wc}$ the angular rate of change of the LOS with respect to the pursuing aircraft. Let the number of time points in an engagement be n. The first fit equations are of the type

$$\overline{\alpha}_i = a\,\overline{wa}_i + b\,\overline{wb}_i + c\,\overline{wc}_i$$

where the constants a, b, and c are chosen so as to minimize

$$E = \sum_{i=1}^{n} (\overline{\theta}_i - \overline{\alpha}_i) \cdot (\overline{\theta}_i - \overline{\alpha}_i)$$

by requiring that $\frac{\delta E}{\delta a} = \frac{\delta E}{\delta b} = \frac{\delta E}{\delta c} = 0$ a, b, and c are readily computed. Some fits of this type attempted were

$$\overline{\alpha_i} = b\ \overline{wb}_i + c\ \overline{wc}_i$$

$$\overline{\alpha_i} = a\ \overline{wa}_i$$

$$\overline{\alpha_i} = b\ \overline{wb}_i$$

$$\overline{\alpha_i} = c\ \overline{wc}_i$$

The second type of fit tried included a bias vector term $\overline{b}$ which was intended to eliminate variations between engagements caused by differing initial conditions at radar lockon. This equation is of the form

$$\overline{\alpha_i} = a\ \overline{wa}_i + \overline{b}$$

where both the constant a and the three constant components of $\overline{b}$, bx, by, and bz must be found. Again, setting

$$\frac{\delta E}{\delta a} = \frac{\delta E}{\delta bx} = \frac{\delta E}{\delta by} = \frac{\delta E}{\delta bz} = 0$$

allows a straightforward computation of these constants.

To obtain a measure of the reduction in lag angle for a given fit, a sample mean and a sample standard deviation calculation is performed on both the lag angle $\overline{\theta}_i$ and on the "unexplained" part of lag angle $\overline{\theta}_i - \overline{\alpha}_i$ as follows:

$$\theta_{av} = \frac{1}{n} \sum_{i=1}^{n} |\overline{\theta}_i|$$

$$\theta_{sd} = \left[\frac{\sum_{i=1}^{n} |\overline{\theta}_i|^2 - n\theta_{av}^2}{n-1} \right]^{\frac{1}{2}}$$

$$fit_{av} = \frac{1}{n} \sum_{i=1}^{n} |\overline{\theta}_i - \overline{\alpha}_i|$$

$$fit_{sd} = \left[\frac{\sum_{i=1}^{n} |\overline{\theta}_i - \overline{\alpha}_i|^2 - n\, fit_{av}^2}{n-1} \right]^{\frac{1}{2}}$$

Fit-Equation Validation

Each of the above fits reduces the average lag angle, sometimes quite dramatically, but of course the key question is "Is there a fit whose constants remain relatively stable from engagement to engagement?" Apparently, there is not. Each of the fits tried exhibits a randomness in its constants over a number of passes even within the same mission.

Of the fits examined, the two giving the best overall reduction in the mean radar lag angle while at the same time having the least variation in the fitting constants were $\overline{\alpha}_i = a\,\overline{wa}_i$, the fit to lag angle as a constant times the true LOS turn rate, and $\overline{\alpha}_i = c\,\overline{wc}_i$, the fit as a constant times the LOS turn rate relative to the pursuing aircraft. The following data are based on 106 passes from the latest tape data. No pass was included that had a mean lag angle in excess of 90 mils. The average lag angle over these passes was 34.8 mils with an average standard deviation of 7.45 mils.

Reference Table 13, list of passes checked, using the fit $\alpha_i = a\,\overline{wa}_i$ gave an average fitted lag angle of 11.8 mils with an average standard deviation of 5.04 mils. But the constant a varied between 0.028 to 0.432 with the bulk of the a's falling between 0.10 and 0.26. The c's varied between the extremes of 0.140 and 0.325. This large variation in fit constants shows clearly the difficulty of attempting to correct for lag angle in this manner. This variation in the two best performing fits tried is a result of the fact that large lag angles occur with both high and low turn rates as do the small lag angles. Assuming

TABLE 13 LIST OF PASSES CHECKED

Mission	Pass	a(sec)	c(sec)	θ_{av}(mils)	Fit_{av} a(mils)	Fit_{av} c(mils)	θ_{sd}(mils)	Fit_{sd} a(mils)	Fit_{sd} c(mils)
76077	1	0.072	0.146	10.7	4.0	5.6	2.3	2.2	2.9
"	2	0.108	0.187	14.9	5.2	5.6	2.8	2.0	2.5
"	3	0.125	0.212	21.6	6.1	6.0	3.3	3.3	1.7
"	4	0.028	0.044	8.3	5.5	5.8	4.0	2.8	3.1
"	5	0.101	0.177	30.2	14.9	7.3	1.6	3.4	2.2
"	6	0.048	0.068	15.9	3.1	4.2	3.0	2.2	3.6
76078	1	0.051	0.129	8.3	5.7	5.6	1.8	2.1	2.2
"	2	0.114	0.220	23.4	7.3	5.7	3.8	3.2	3.5
"	3	0.041	0.294	15.9	14.9	11.8	4.5	5.1	4.7
"	4	0.138	0.419	17.5	3.4	6.4	3.1	1.5	4.5
"	5	0.029	0.215	18.8	18.3	15.3	13.7	13.9	11.3
"	6	0.159	0.228	47.5	9.3	8.0	8.9	4.7	2.6
"	7	0.120	0.180	35.9	17.5	12.5	3.2	5.0	3.1
"	8	0.170	0.223	45.0	15.3	8.6	1.9	4.0	3.9
76083	1	0.130	0.183	37.5	4.1	9.2	3.7	1.4	1.7
"	2	0.113	0.140	30.2	15.8	14.2	4.8	2.8	2.0
"	3	0.120	0.150	33.7	1.7	4.9	2.0	1.2	0.5
"	4	0.158	0.217	37.6	15.2	13.7	2.3	1.0	1.1
76085	1	0.054	0.555	26.5	24.4	13.6	4.7	8.5	6.1
76092	1	0.240	0.248	35.0	21.1	29.5	14.7	13.7	12.1
76093	1	0.082	0.365	25.1	22.0	11.3	7.4	9.0	4.9
76110	1	0.432	0.728	87.1	50.5	59.2	27.2	21.7	19.3
"	4	0.366	0.561	82.2	41.0	29.6	39.0	14.4	12.5
"	6	0.422	0.851	78.9	41.8	19.5	39.4	18.6	9.3
76111	1	0.134	0.161	23.2	5.5	6.3	4.0	2.2	2.8
"	2	0.171	0.246	21.8	7.1	8.2	1.6	1.5	1.3
"	3	0.145	0.202	40.6	12.9	4.6	3.6	2.4	1.4
"	4	0.130	0.180	36.8	6.8	4.5	5.9	3.4	2.1
"	5	0.149	0.229	45.8	14.7	2.0	5.0	1.8	0.6
76118	1	0.188	0.243	28.8	3.9	4.2	2.6	2.1	1.8
"	2	0.170	0.191	30.8	6.5	9.8	1.5	3.4	3.8
"	3	0.184	0.258	42.0	8.5	5.6	4.5	3.6	3.0
"	4	0.067	0.084	15.4	5.4	5.2	2.5	3.4	3.5
"	5	0.174	0.207	33.6	3.8	7.2	1.8	1.6	2.0
"	6	0.251	0.301	74.9	14.7	8.7	12.1	4.7	4.7
"	7	0.221	0.251	66.5	12.1	14.3	2.9	6.7	1.2
76119	2	9.120	0.306	11.8	4.7	4.4	3.3	2.0	3.1
"	3	0.114	0.168	16.5	5.1	7.2	3.7	2.8	4.7
"	4	0.166	0.278	25.8	4.3	7.5	1.9	1.3	3.2
"	5	0.097	0.312	20.6	16.3	14.0	6.5	8.0	5.6
"	6	0.201	0.291	36.4	7.6	17.0	2.7	3.2	10.2
"	7	0.223	0.306	45.9	9.7	7.2	3.0	6.3	2.7
76121	1	0.037	0.088	5.7	4.9	4.7	3.2	2.4	2.3
"	2	0.069	0.160	11.8	9.8	9.7	3.0	2.5	1.5
"	3	0.080	0.243	10.7	7.3	6.4	1.6	1.5	2.1
"	4	0.079	0.215	14.6	7.2	6.7	4.5	4.7	4.8
"	5	0.236	0.282	50.3	4.5	9.8	3.5	1.2	1.1
"	6	0.132	0.163	32.3	8.3	3.9	2.4	1.9	0.8
"	7	0.119	0.198	24.1	6.1	13.9	2.6	1.6	2.0
"	8	0.072	0.075	17.6	12.4	7.5	1.9	2.8	2.8
"	9	0.202	0.256	55.1	7.1	5.8	5.7	2.5	3.0
76126	1	0.066	0.210	12.2	8.3	18.7	1.5	1.9	1.4
"	2	0.135	0.179	24.3	18.0	7.4	1.5	3.4	1.8
"	3	0.113	0.246	11.5	7.6	18.3	3.9	2.3	2.9

TABLE 13 LIST OF PASSES CHECKED (CONTINUED)

Mission	Pass	a(sec)	c(sec)	θ_{av}(mils)	Fit_{av} a(mils)	Fit_{av} c(mils)	θ_{sd}(mils)	Fit_{sd} a(mils)	Fit_{sd} c(mils)
76131	1	0.115	0.243	19.4	18.2	18.3	21.4	15.2	15.7
"	2	0.042	0.050	14.9	14.2	14.5	2.1	2.6	2.5
"	3	0.166	0.302	47.8	14.1	18.2	17.1	6.2	6.9
"	4	0.105	0.253	24.1	14.7	11.6	5.0	3.1	3.3
"	5	0.107	0.171	28.4	9.8	6.0	2.9	2.2	2.1
"	6	0.059	0.206	12.1	8.3	6.4	2.8	2.8	2.9
"	7	0.194	0.305	58.8	19.5	6.8	5.8	1.4	4.6
"	8	0.229	0.345	72.5	14.2	18.0	12.1	4.4	2.6
"	10	0.059	0.091	21.6	15.4	15.8	4.8	2.3	2.0
76162	1	0.065	0.243	81.0	76.7	71.0	60.0	63.5	65.6
"	2	0.143	0.278	44.1	22.7	15.1	16.7	4.0	2.2
"	3	0.151	0.205	40.0	14.7	12.6	5.2	3.5	2.8
"	4	0.120	0.281	36.8	21.3	20.2	23.9	27.6	18.0
"	5	0.198	0.243	66.3	16.9	18.1	8.7	1.9	1.4
"	6	0.151	0.326	61.2	32.5	14.1	15.3	1.8	1.7
"	7	0.194	0.225	65.4	54.5	49.8	58.7	31.2	63.3
76163	1	0.260	0.386	34.5	12.4	11.3	2.8	4.7	4.3
"	3	0.119	0.209	36.1	9.7	3.1	2.3	3.3	1.5
"	4	0.249	0.306	49.0	9.8	10.1	3.2	3.7	3.1
"	5	0.293	0.376	42.8	9.7	10.2	4.2	4.0	2.9
"	6	0.169	0.201	51.3	9.2	6.1	4.8	5.0	3.1
"	7	0.069	0.146	14.5	11.1	10.0	2.4	2.5	3.2
"	8	0.251	0.312	52.3	8.2	4.8	4.3	4.7	3.4
"	9	0.260	0.285	48.9	10.4	13.2	5.4	3.8	3.1
76166	1	0.031	0.174	13.8	13.5	10.0	6.2	5.6	4.4
"	2	0.032	0.183	5.7	4.9	3.5	3.1	2.9	1.4
"	3	0.119	0.144	22.6	11.0	12.7	4.9	4.8	5.2
"	5	0.092	0.216	26.6	20.7	15.6	6.4	5.4	7.2
"	6	0.070	0.128	25.2	12.4	5.5	3.9	5.1	2.7
"	7	0.059	0.112	15.2	6.5	4.0	2.0	3.0	2.0
"	8	0.108	0.185	31.9	10.6	5.6	3.8	1.9	2.0
76174	1	0.055	0.130	16.6	15.3	14.4	3.7	3.7	3.0
"	2	0.234	0.257	82.5	11.9	8.4	5.4	3.8	3.3
76175	2	0.103	0.187	46.2	35.3	29.6	3.9	8.8	4.9
"	3	0.088	0.102	23.4	17.8	19.0	4.8	6.1	4.5
"	4	0.151	0.181	54.2	28.4	27.8	11.1	2.6	3.5
"	6	0.128	0.152	28.4	10.9	10.7	4.9	6.3	3.6
"	7	0.191	0.223	57.1	31.9	32.3	2.6	1.8	2.3
"	8	0.124	0.187	45.5	13.8	3.7	4.1	1.4	2.0
"	9	0.151	0.245	49.8	20.0	4.3	3.6	3.6	2.0
76197	1	0.046	0.074	8.8	3.6	2.8	2.7	2.0	1.3
75338	1	0.125	0.304	32.9	21.5	9.7	3.7	3.4	3.4
"	2	0.070	0.105	15.1	2.8	4.3	2.8	1.4	1.8
"	3	0.115	0.277	33.1	26.1	20.6	4.6	5.9	2.7
"	4	0.094	0.171	21.9	10.4	10.0	14.3	14.0	10.8
76162	1	0.160	0.303	39.5	17.2	15.5	9.5	13.8	9.0
"	2	0.165	0.194	45.2	12.2	8.0	17.3	3.9	3.1
"	3	0.172	0.226	43.3	7.9	5.6	5.3	2.7	2.1
"	4	0.157	0.287	34.7	8.9	10.4	14.5	8.1	7.7
"	5	0.218	0.259	68.0	7.3	9.9	11.6	2.9	2.9
"	6	0.159	0.317	58.4	20.1	5.9	15.0	3.9	2.8
"	7	0.150	0.268	41.3	9.0	6.6	14.2	6.2	5.2

that the Sight Eval data does indeed present a substantially true picture of the system performance, the data conveys a strong impression of erratic system performance.

It seems reasonable to conclude that whatever the causes of the radar lag problem, the lag angle is not a function of the engagement geometry. This conclusion can be criticized on the basis that a higher order fit should have been tried or some conceptually different type of fit involving different engagement variables might have been successful, etc. Perhaps. But the fact remains that the various turn rates used all have roughly the same direction as the vector lag angle and most of the engagement variables are involved in computing these turn rates. There should have been some perceivable trend in the fit constants somewhere. After all, the lag angle problem is caused largely by an incorrect axis turn rate with perhaps an initial misorientation problem added for good measure.

There does exist the possibility of taking one of the fits and working up those values of its constants which yield the best overall statistical decrease in radar lag angle for a large number of engagements. But there are two obvious hazards in the procedure. First, if the true source of the problem remains undiscovered, what assurance is there that there will not be radical changes in system performance in the future? Second, since the engagements are so similar and so brief, would the correction be valid in engagements having a different geometry or a greater duration?

One further comment is necessary. It may seem that the more direct approach of doing a system analysis, on the MA-1 system and its subsequent modifications, was ignored. After all, the original applications of the MA-1 system radar did not require it to produce the highly accurate target angular direction data now being asked of it. An analysis of the MA-1 system design and performance was initiated. This analysis required detailed system design theory and performance data as well as systems

modification and modified system performance data of the MA-1 used in Sight Eval testing. The extensive period since the original design and test of the MA-1 materially reduced the availability of pertinent design and performance data, leaving the geometrical approach as the only analysis alternative.

3.3.4.3 Time Data Phase Shift

An examination of the plots of radar antenna position in azimuth and elevation versus film target position shows an apparent data phase shift which varies up to .5 seconds and averages approximately .25 seconds. There are a number of reasons why such a phase shift could exist. Delay in instrumentation circuitry is a function of the number of processing and amplification stages through which an electronic signal must pass. However, this category of delay results in less than a microsecond per stage and will seldom reach the .25 second delay observed. A more likely possibility is a discrepancy in the synchronization of magnetic tape and film data during the merger into a single data tape. Of course, the probability is high that the phase shift does indeed exist, and, if so, the direct modification to the correction algorithm is obvious. Graphs of passes that illustrate the data shift are shown in Figures 33 thru 37.

The most obvious example of a data shift is shown in Figure 33. On this pass the radar was leading the film data in azimuth. This immediately suggests a data merging or calibration problem as the radar can never constantly lead the target. The occurrence of radar lead was not often obvious, but Figure 34 strongly suggests that the radar was leading the elevation for approximately 1.1/2 of the 3 seconds of available data. The radar data follows the film data profile very closely and the differences between film and radar are small, and rarely exceed 10 MRAD. If not closely inspected, this pass would have been accepted as an ideal pass.

The third example of a possible data shift in azimuth is seen in Figure 35. The elevation radar data closely follows the film data with the radar occasionally overshooting the target, but immediately being corrected. This is how one would expect the radar to behave, and the component differences rarely exceed 15 MRAD. Inspection of the azimuth data reveals a

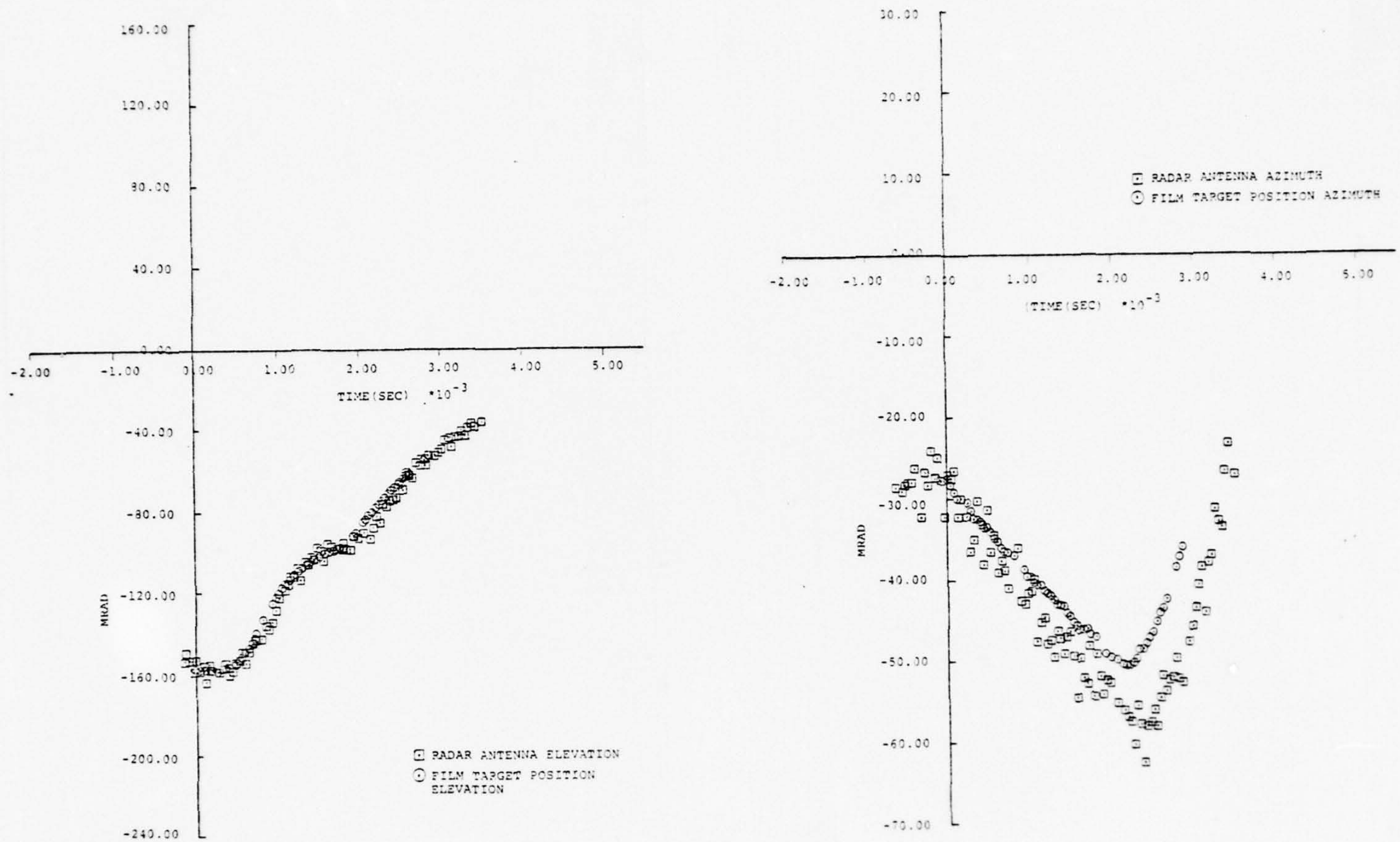


Figure 33 Radar Leading Target

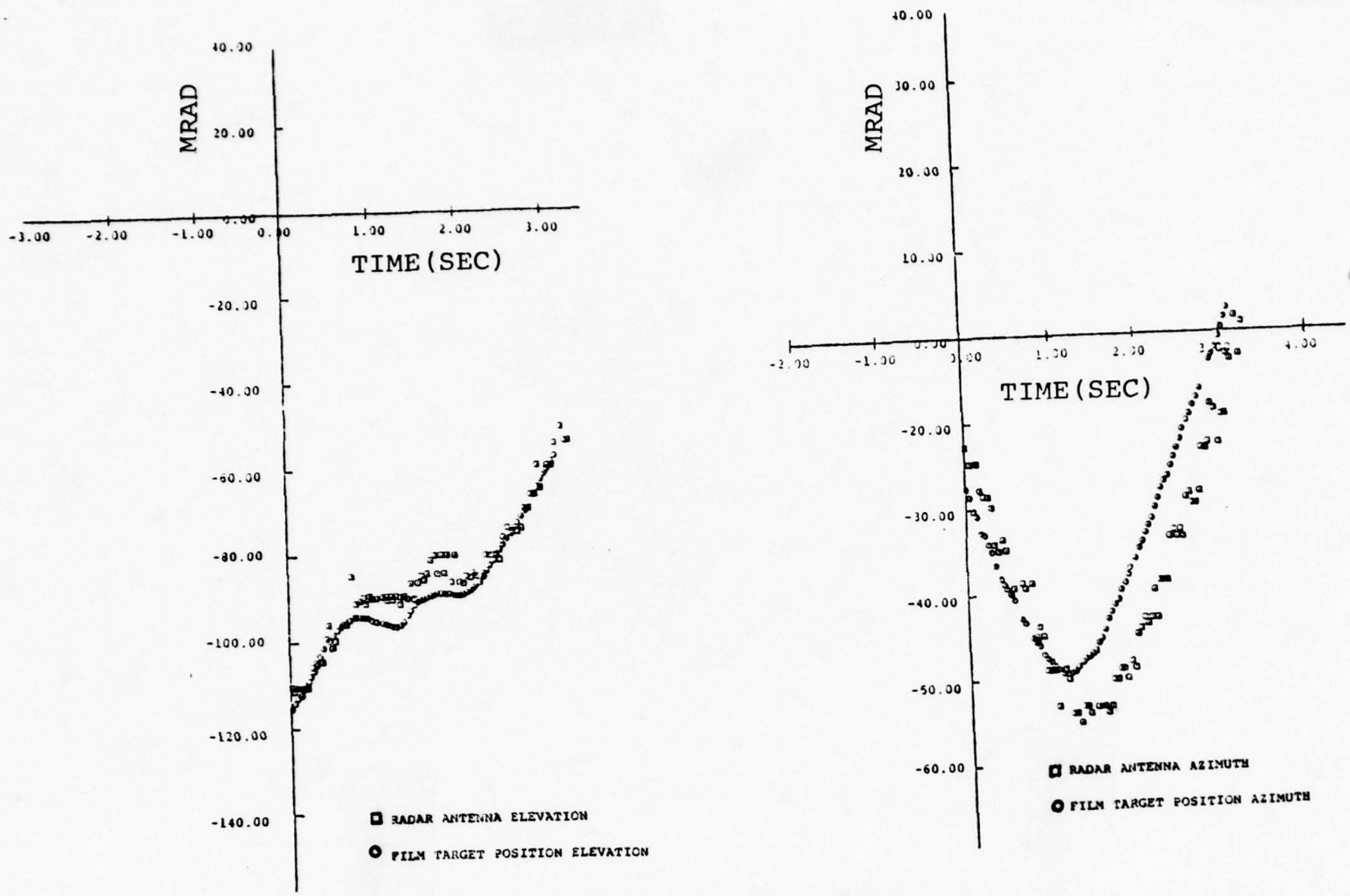


Figure 34 Radar Leading Target

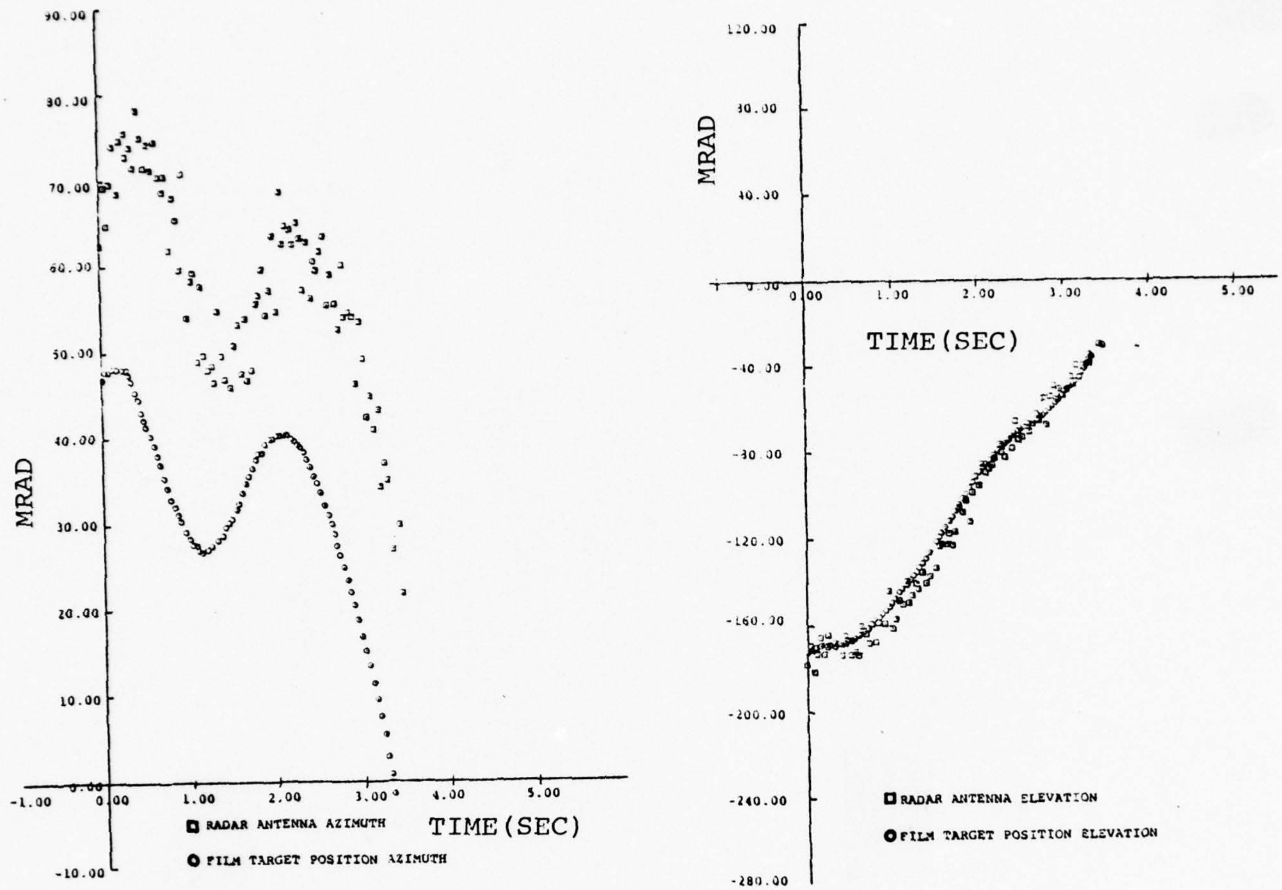


Figure 35 Lag in Radar Azimuth

very poor tracking job by the radar. The same basic profile is observed by radar and film but the radar is lagging the film by approximately 25-30 MRAD. Also the radar profile, if smoothed, seems to be about 1/4 second out of synchronization with the film.

The above examples are extremes noted in reviewing the data. The largest percentage of the passes showed the radar lagging the film data. This lag time was felt to be excessive in most cases. The radar profiles in Figures 36 and 37 suggest the radar is roughly 1/4 second behind the film data and are typical of many passes. Being unable to isolate the true radar lag from a data shift, it was decided to try a variety of time shifts. It was felt that the radar should respond with a correction in less than a tenth of a second. Accordingly, time corrections of 4 to 8 frames were selected to see if the total lag angle could be consistently corrected.

Program SDATA described in detail in Section 3.3.4.2 was modified to perform all lag angle calculations in the program inertial coordinate system, with the film target direction being transformed to inertial coordinates by that transformation consistent with the lag time assumed. Two tapes were run with this correction. The results of the data shift are shown in Tables 14 and 15.

In many passes the lag angle is reduced below the 15 MRAD requirement. In Table 14, 5 of the 29 passes had lag angles less than 15 MRAD. After the frame shift, 24 of 29 had lag angles less than 15 MRAD. In Table 15, 4 of the 26 passes had lag angles less than 15 MRAD. After the frame shift, 14 of 24 had lag angles less than 15 MRAD. Of the 54 passes tried for this data shift, only 2 that would have been accepted were made unacceptable by the shift. However, there appears to be a definite contribution to lag angle from this data shift. Further, an error in synchronization is the only possible explanation for those few passes which show the radar leading the film.

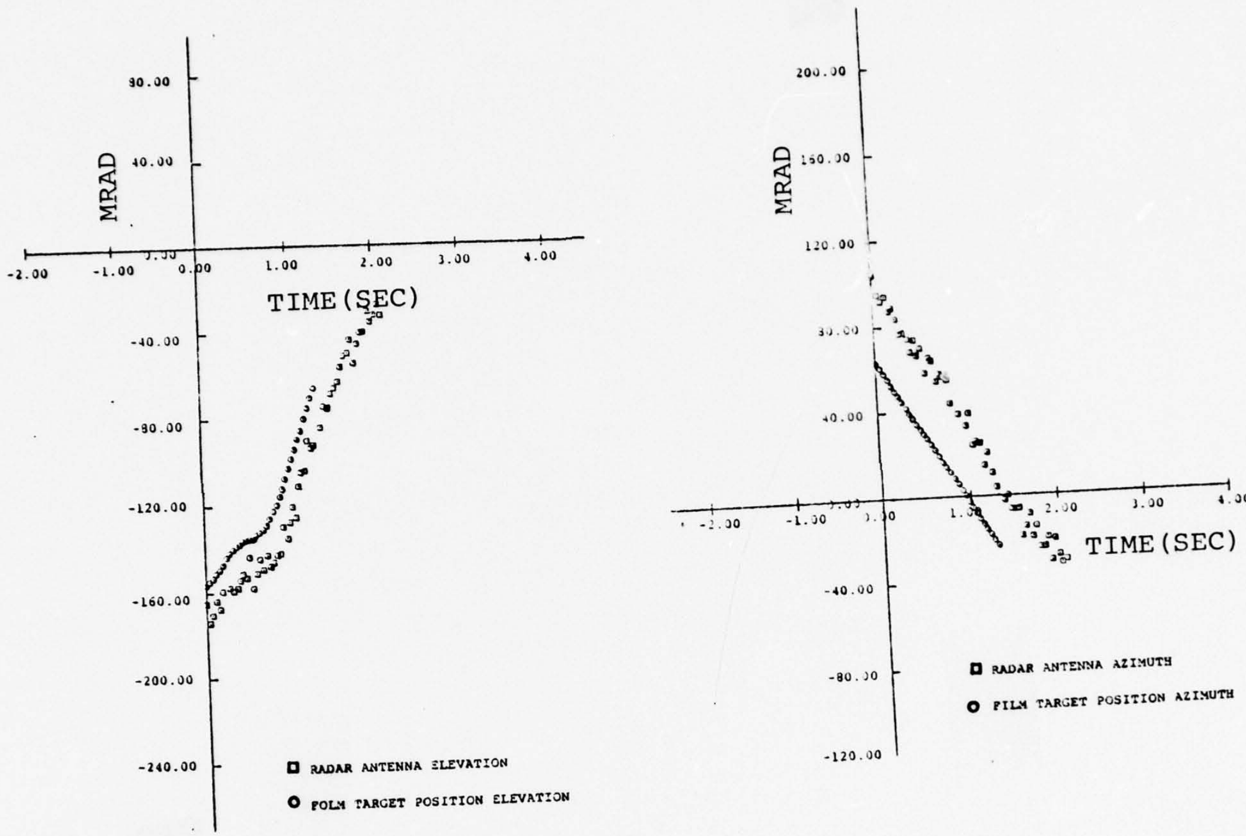


Figure 36 Lag in Both Radar Components

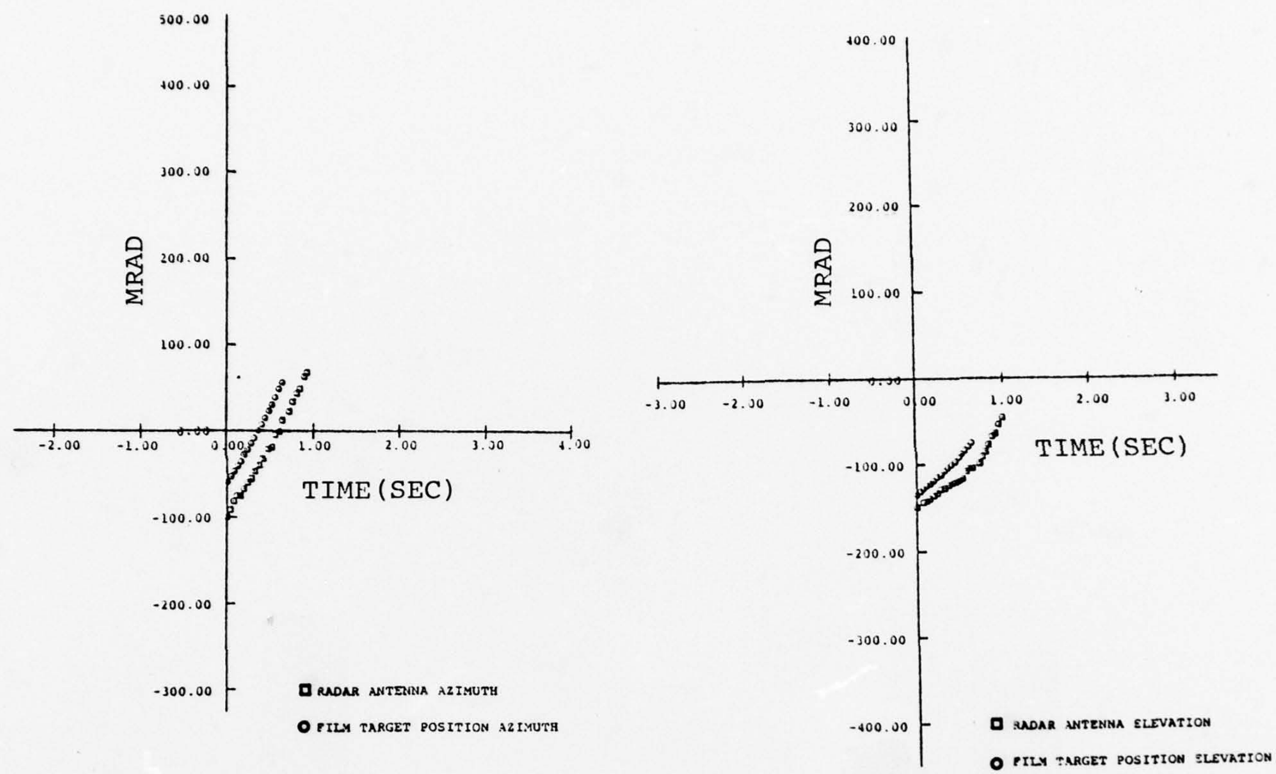


Figure 37 Lag in Both Radar Components

TABLE 14 Comparison of Average Lag Before and After Frame Shift

MISSION	PASS	AVERAGE LAG MRAD	BEST FRAME SHIFT (No of Frames)	ADJUSTED LAG MRAD	FRAMES ERROR < 15 MRAD	COMMENTS
76111	1	23.2	4	6.12	4-6	
	2	21.8	6	7.73	4-8	
	3	40.6	5	4.50	4-7	
	4	36.8	5	4.34	4-6	
	5	45.8	6	2.07	5-7	
76118	1	28.8	6	3.93	4-8	
	2	30.8	5	6.65	4-6	
	3	42.0	6	5.44	5-8	
	4	15.4	4	14.89	4	
	5	33.6	5	7.23	4-6	
	6	74.9	6	5.97	5-6	
	7	66.5	6	15.03	---	
	8	135.5	8	130.48	---	No Radar Lock
76119	1	383.5	4	388.14	---	No Radar Lock
	2	11.8	8	4.02	4-8	
	3	16.5	4	6.18	4-7	
	4	25.8	6	7.96	4-8	
	5	20.6	6	16.81	---	
	6	36.4	8	12.85	7-8	
	7	45.9	6	5.59	5-8	
76121	1	5.7	4	7.28	4-8	
	2	11.8	4	10.11	4-8	
	3	10.7	6	6.79	4-8	
	4	14.6	4	9.99	4-6	
	5	50.3	6	6.71	5-8	
	6	32.3	4	11.29	4	
	7	24.1	4	3.92	4-6	
	8	17.6	4	19.33	---	
	9	55.1	6	4.57	5-8	

TABLE 15 Comparison of Average Lag Before and After Frame Shift

MISSION	PASS	AVERAGE LAG MRAD	BEST FRAME SHIFT (No of Frames)	ADJUSTED LAG MRAD	FRAMES ERROR < 15 MRAD	COMMENTS
76077	1	10.7	4	6.19	4-8	
	2	14.9	4	6.34	4-8	
	3	21.6	5	10.33	4-6	
	4	8.3	4	18.75	---	
	5	30.2	5	7.88	4-6	
	6	15.9	4	25.61	---	
76078	1	8.3	4	6.13	4-8	
	2	23.4	5	6.25	4-8	
	3	15.9	4	14.73	4-6	
	4	17.5	8	8.19	4-8	
	5	18.8	4	16.02	---	
	6	47.5	7	8.99	5-8	
	7	35.9	5	11.92	4-6	
	8	45.0	6	8.07	4-7	
76083	1	37.5	5	8.81	4-7	
	2	30.2	4	16.64	---	
	3	33.7	4	6.45	4-5	
	4	37.6	5	13.88	4-5	
76085	1	26.5	7	20.78	---	
76092	1	35.0	4	34.22	---	
	2	13.01	4	24.07	---	Radar Lead (Elevation)
76093	1	25.1	8	15.52	---	
	2	63.5	4	78.92	---	No Radar Lock
	3	185.3	8	181.59	---	No Radar Lock
	4	226.6	8	225.84	---	No Radar Lock

Again reaching a general conclusion is made difficult by the lack of detailed documentation on the experimental design and on the data reduction procedures. This procedure did not give a great improvement in all passes, but the results do indicate that the radar may be considerably more accurate in determining target angular direction, than was previously thought.

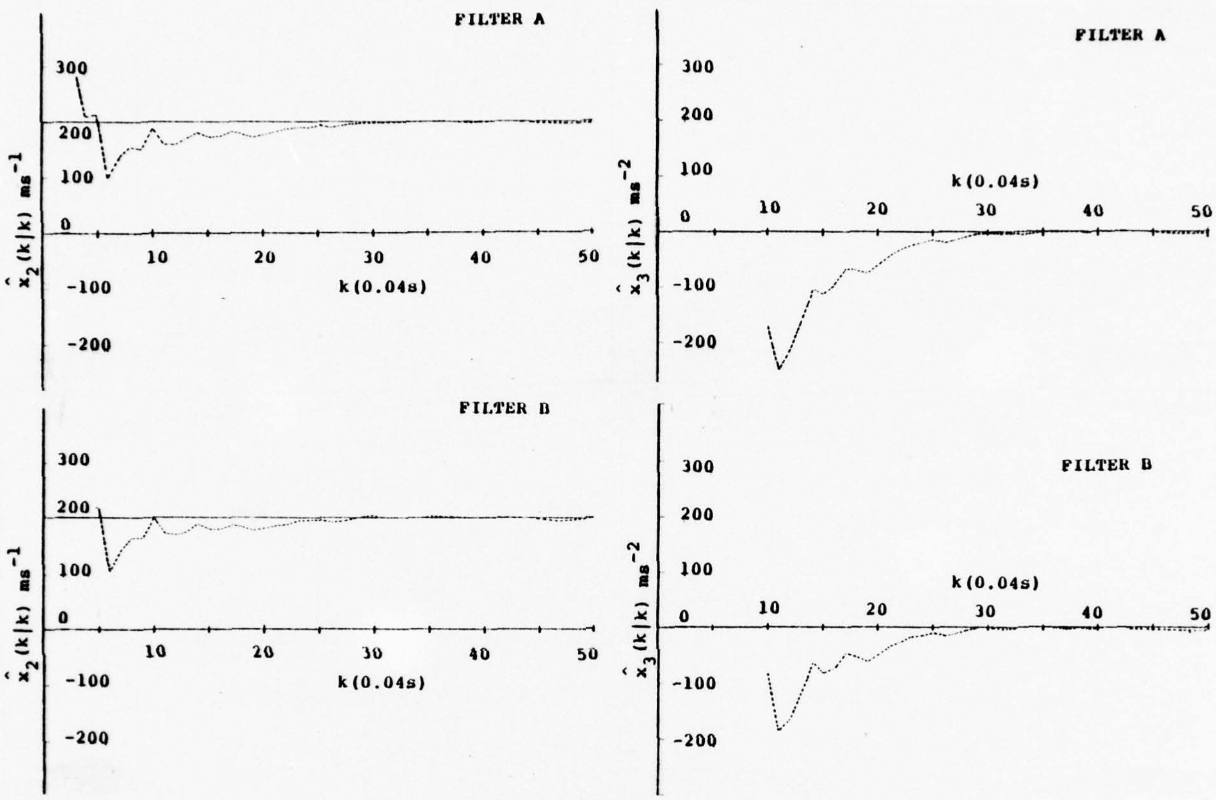
3.3.4.4 Suggestions for Future Study

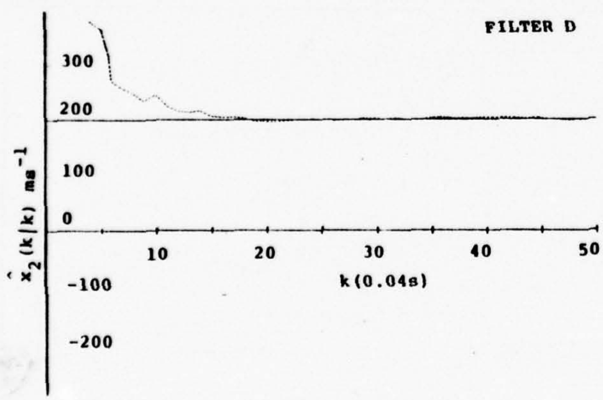
It is evident that a time lag occurred between the times the film and radar data were recorded. This lag may be an actual interval introduction by real radar error or it may be an artificial interval caused by instrumentation or data merger into a single tape. This time lag which results in radar error is of sufficient importance that a follow-on study should determine the true source of the delay.

In connection with the current analysis, the availability of a detailed instrumentation and test plan, maintenance, calibration, and modification logs for the aircraft and fire control system, and details of data processing would have been of material assistance. In future testing, the close coordination between analysis and test planning should be emphasized, with consideration given to a single analysis/Test Manager. A sample analysis exercise concurrently with initial flight tests will eliminate many later analysis snags.

APPENDIX A

COMPARISON OF KALMAN FILTERS



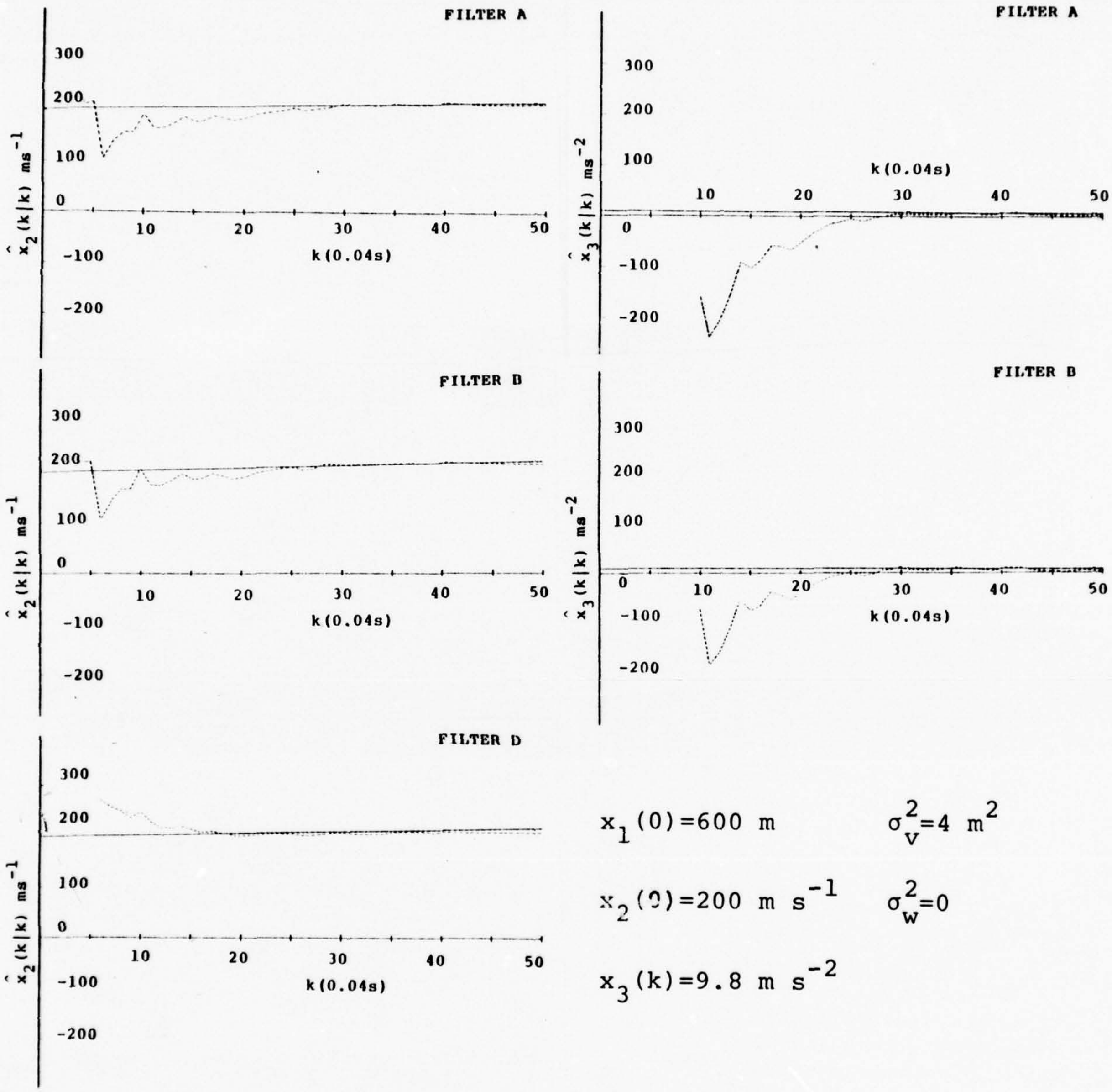


$x_1(0)=600$ m $\qquad \sigma_v^2=4$ m^2

$x_2(0)=200$ m s^{-2} $\qquad \sigma_w^2=0$

$x_3(k)=0$

Figure A-1

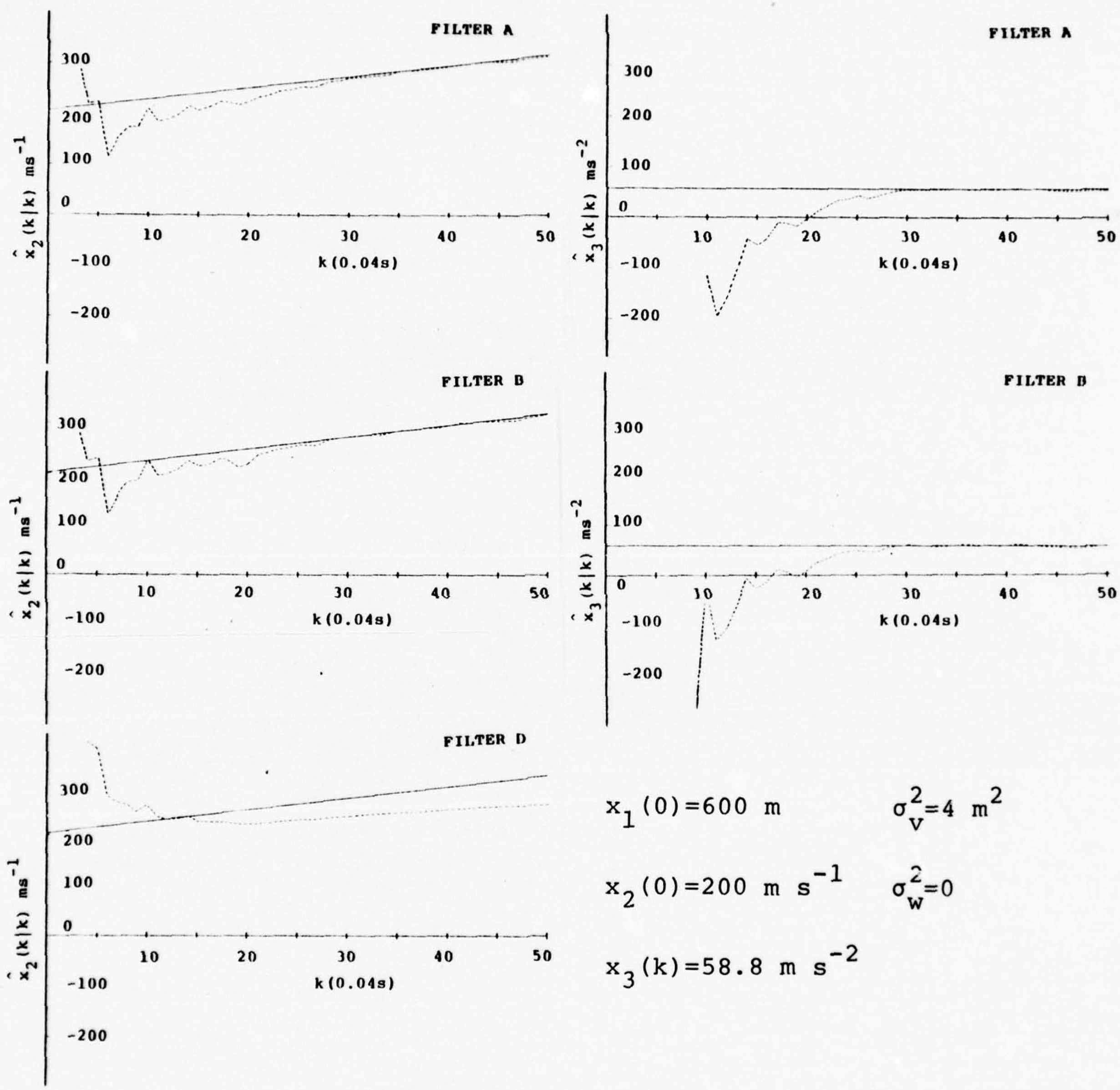


$x_1(0)=600$ m $\quad \sigma_v^2=4$ m^2

$x_2(0)=200$ m s^{-1} $\quad \sigma_w^2=0$

$x_3(k)=9.8$ m s^{-2}

Figure A-2

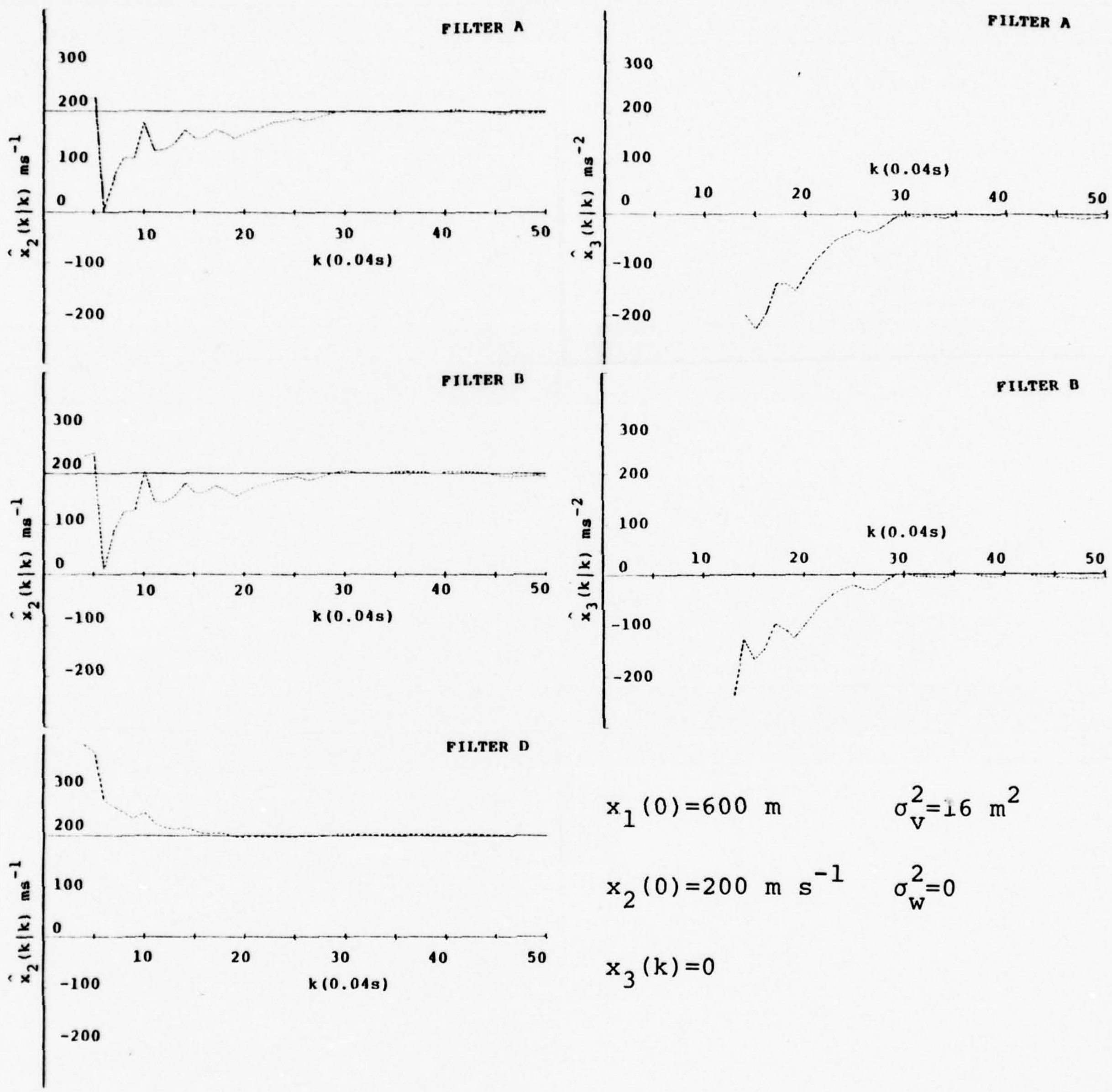


$x_1(0)=600$ m $\quad \sigma_v^2=4$ m^2

$x_2(0)=200$ m s^{-1} $\quad \sigma_w^2=0$

$x_3(k)=58.8$ m s^{-2}

Figure A-3



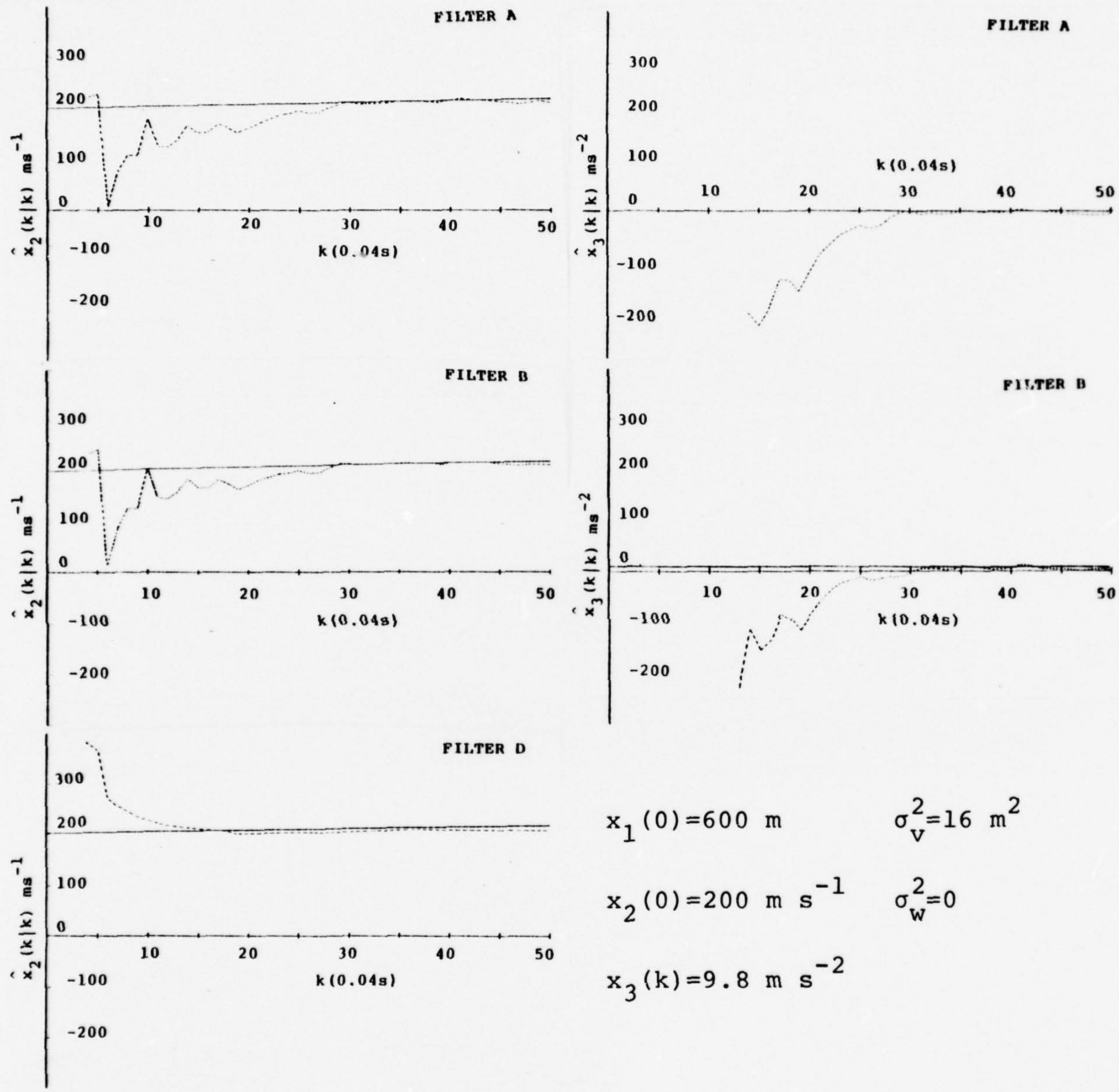
$x_1(0)=600$ m $\quad \sigma_v^2=16$ m^2

$x_2(0)=200$ m s^{-1} $\quad \sigma_w^2=0$

$x_3(k)=0$

Figure A-4

5

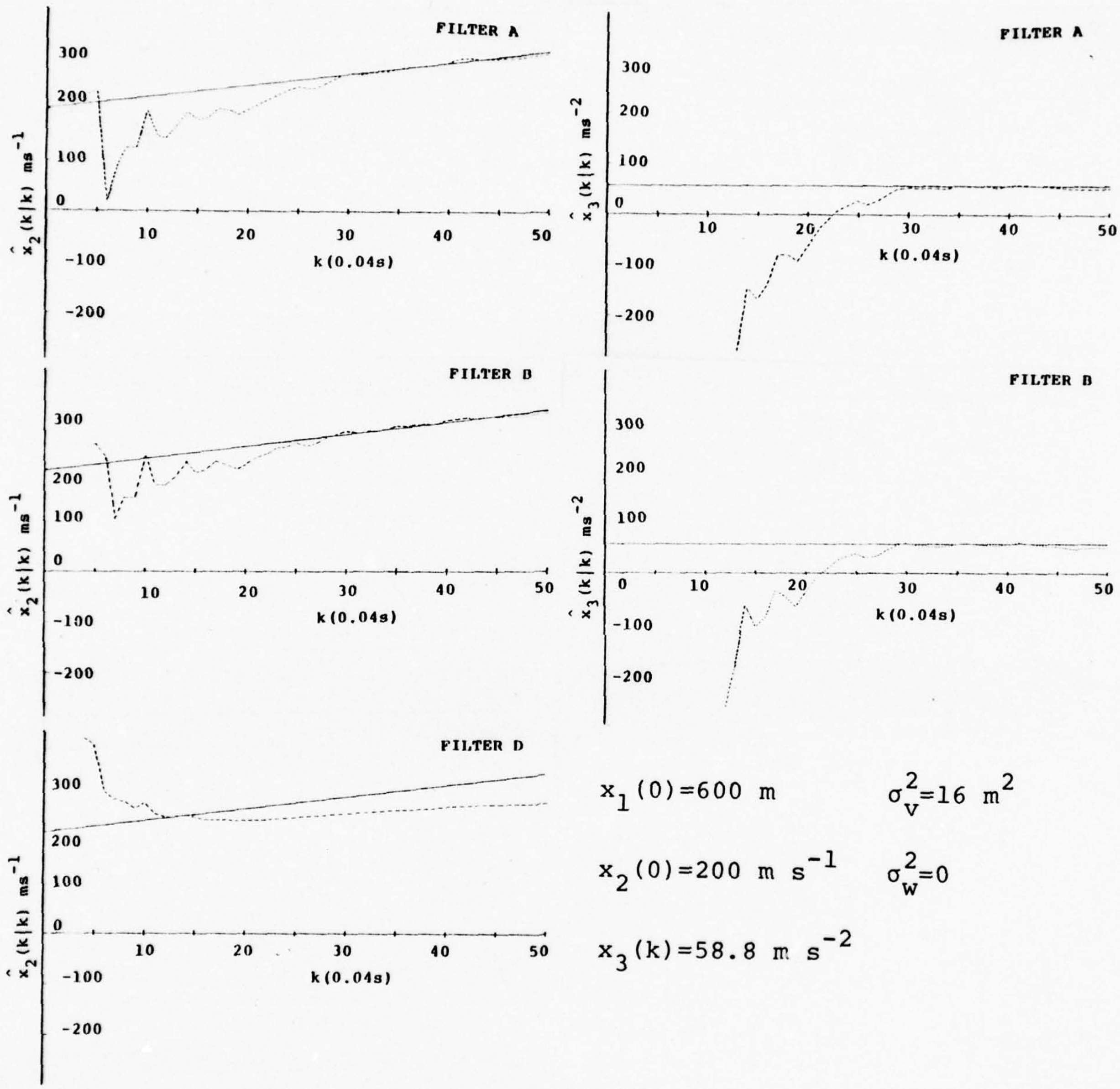


$x_1(0)$=600 m σ_v^2=16 m^2

$x_2(0)$=200 m s^{-1} σ_w^2=0

$x_3(k)$=9.8 m s^{-2}

Figure A-5



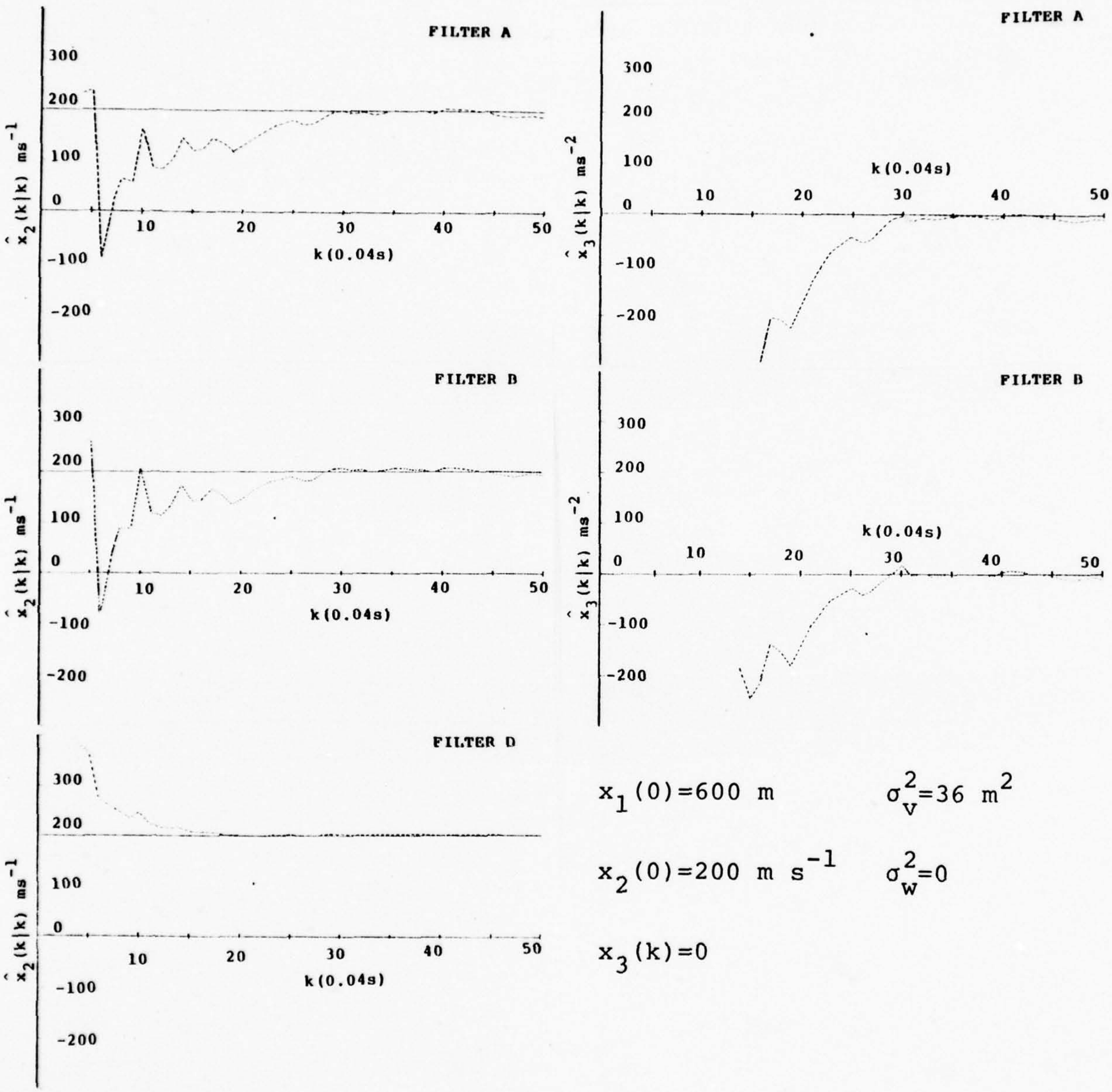
$x_1(0)=600$ m $\qquad \sigma_v^2=16$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=0$

$x_3(k)=58.8$ m s^{-2}

Figure A-6

7

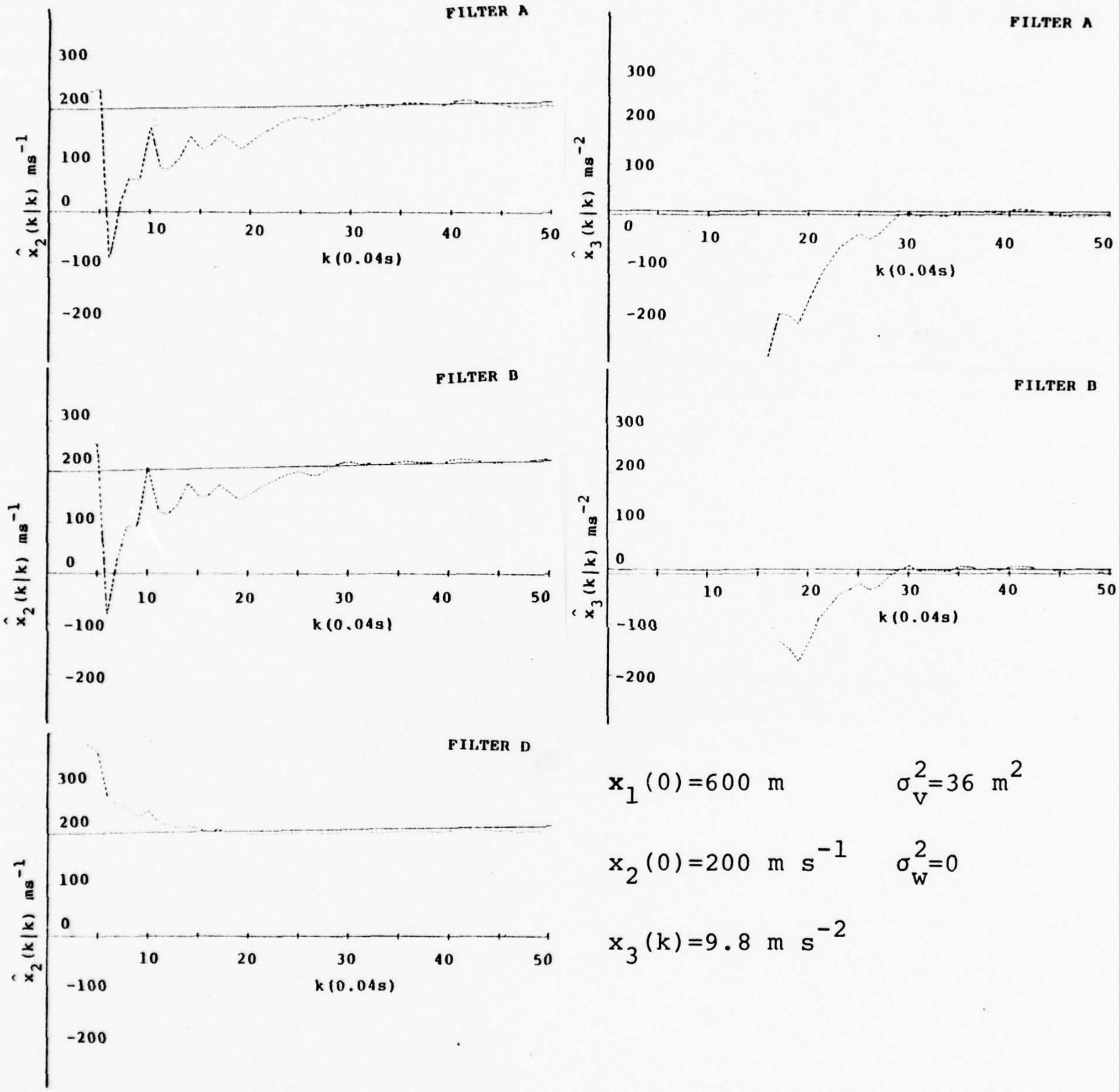


$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=0$

$x_3(k)=0$

Figure A-7



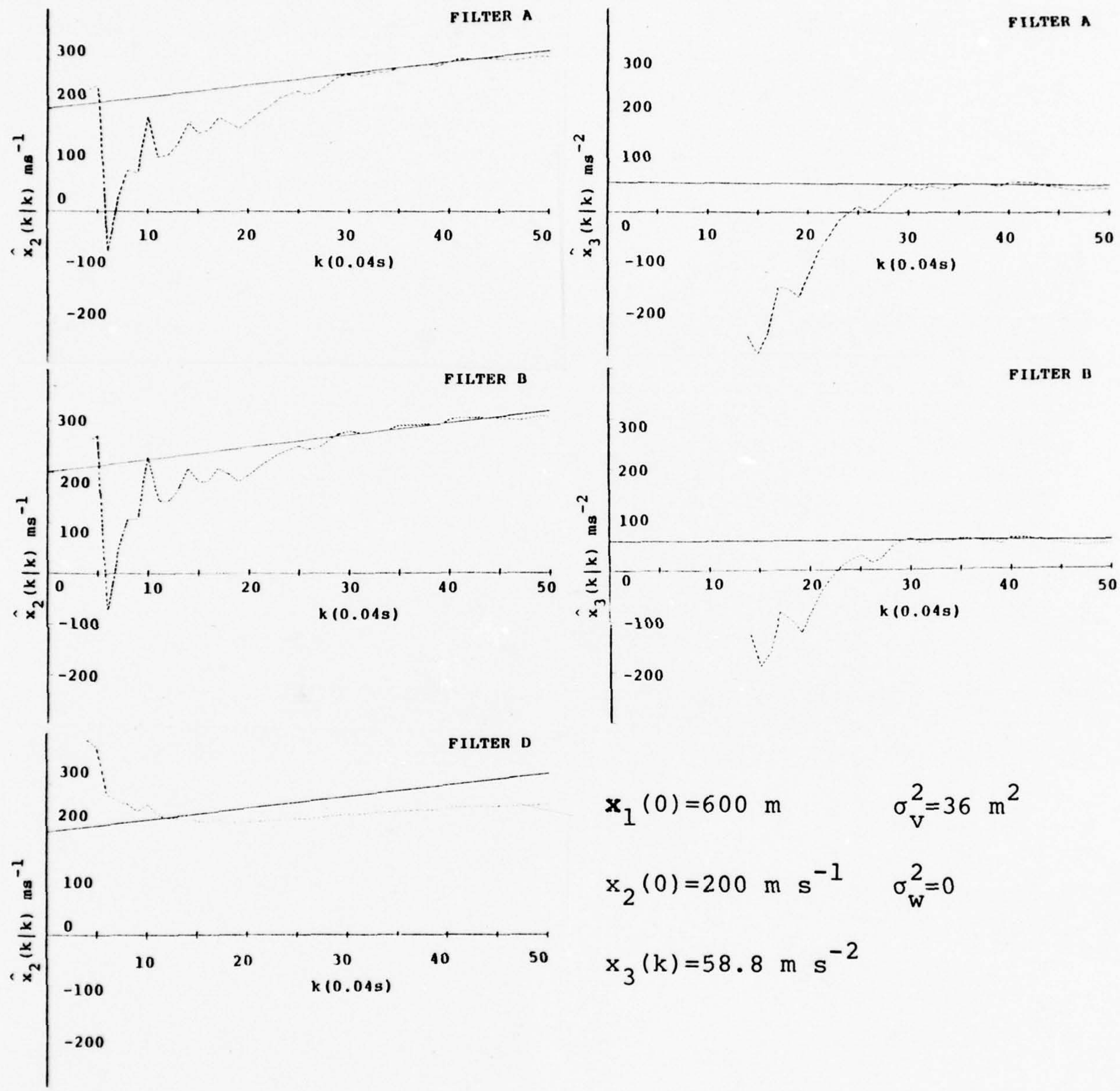
$x_1(0)=600$ m $\quad \sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\quad \sigma_w^2=0$

$x_3(k)=9.8$ m s^{-2}

Figure A-8

9



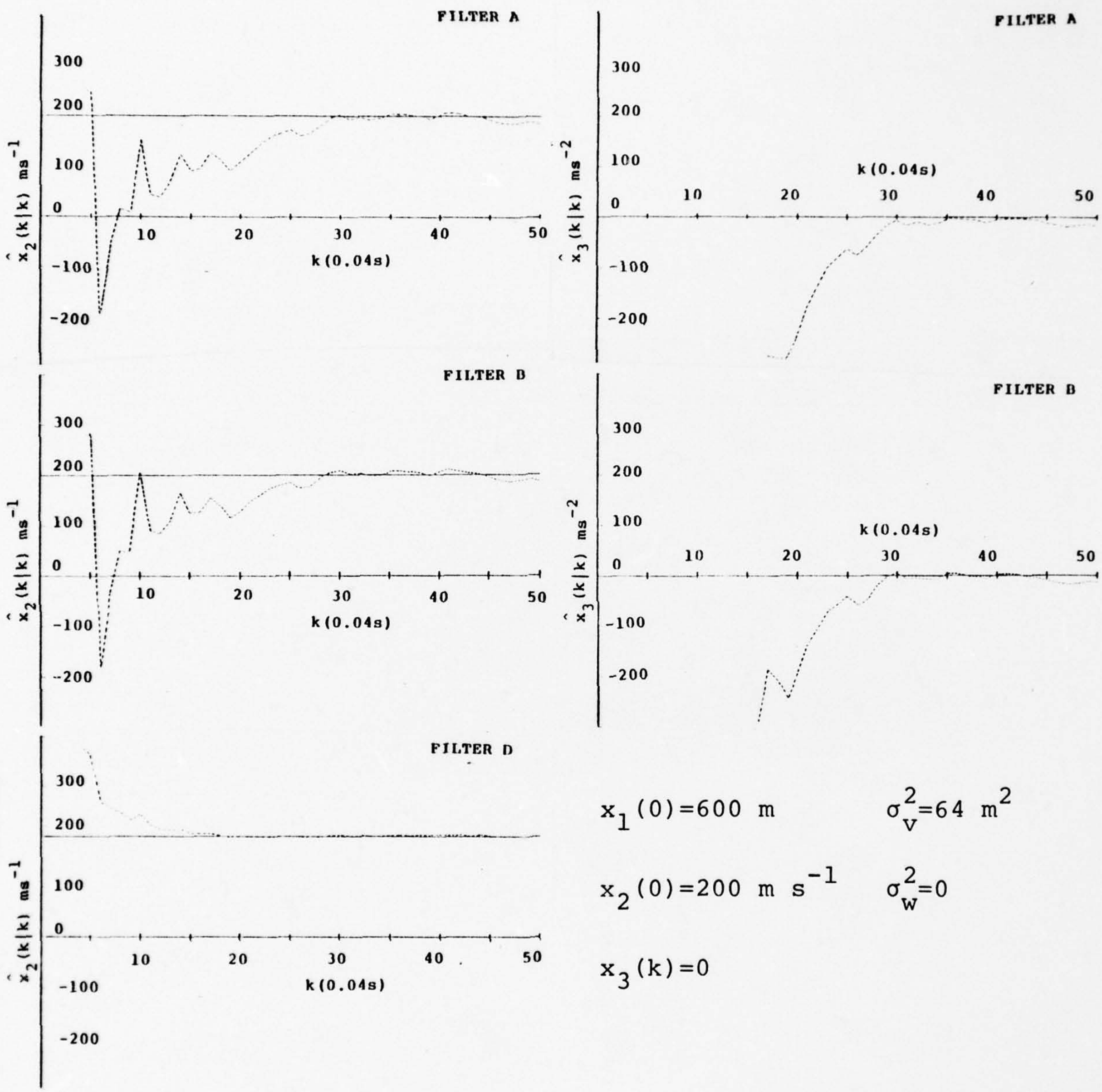
$x_1(0)=600$ m $\quad \sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\quad \sigma_w^2=0$

$x_3(k)=58.8$ m s^{-2}

Figure A-9

10

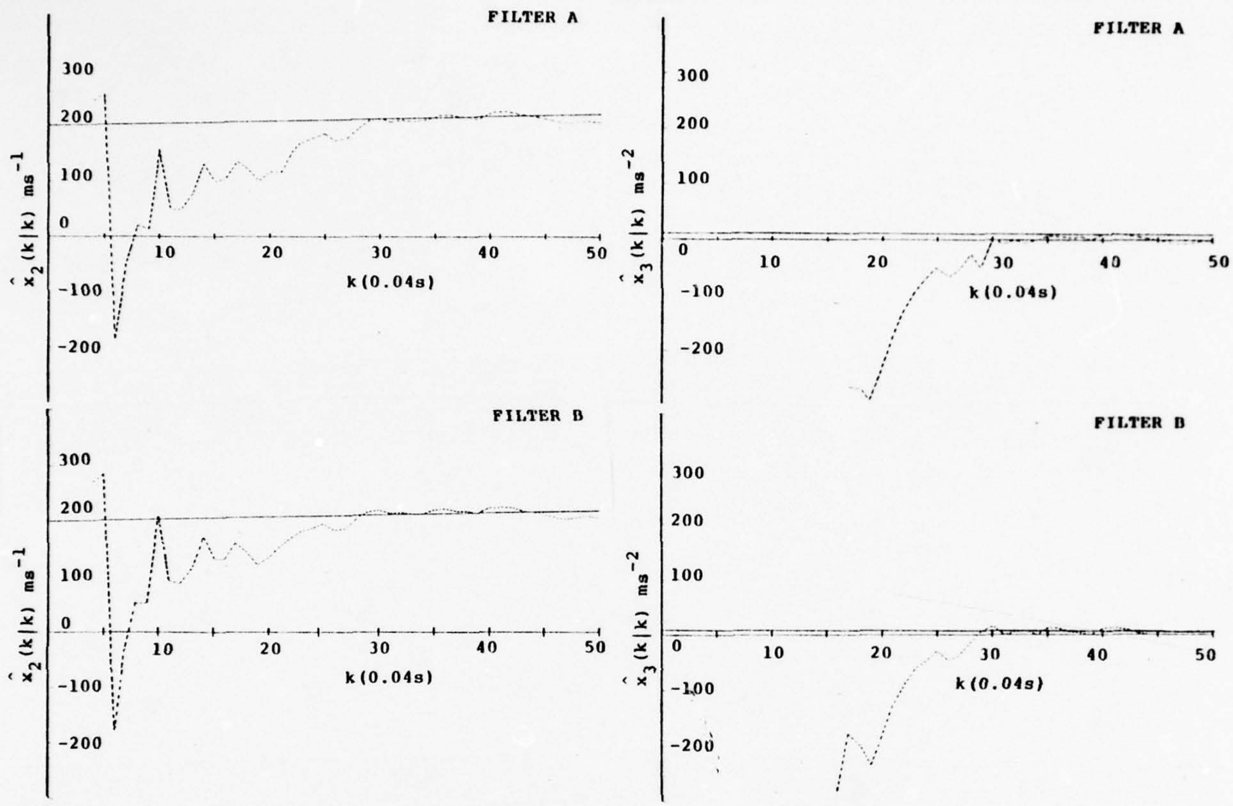


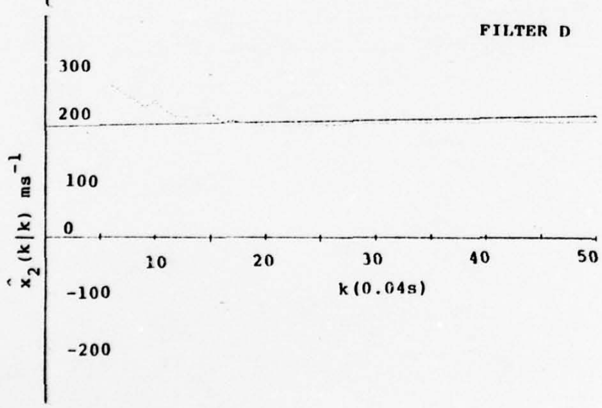
$x_1(0)=600$ m $\qquad \sigma_v^2=64$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=0$

$x_3(k)=0$

Figure A-10



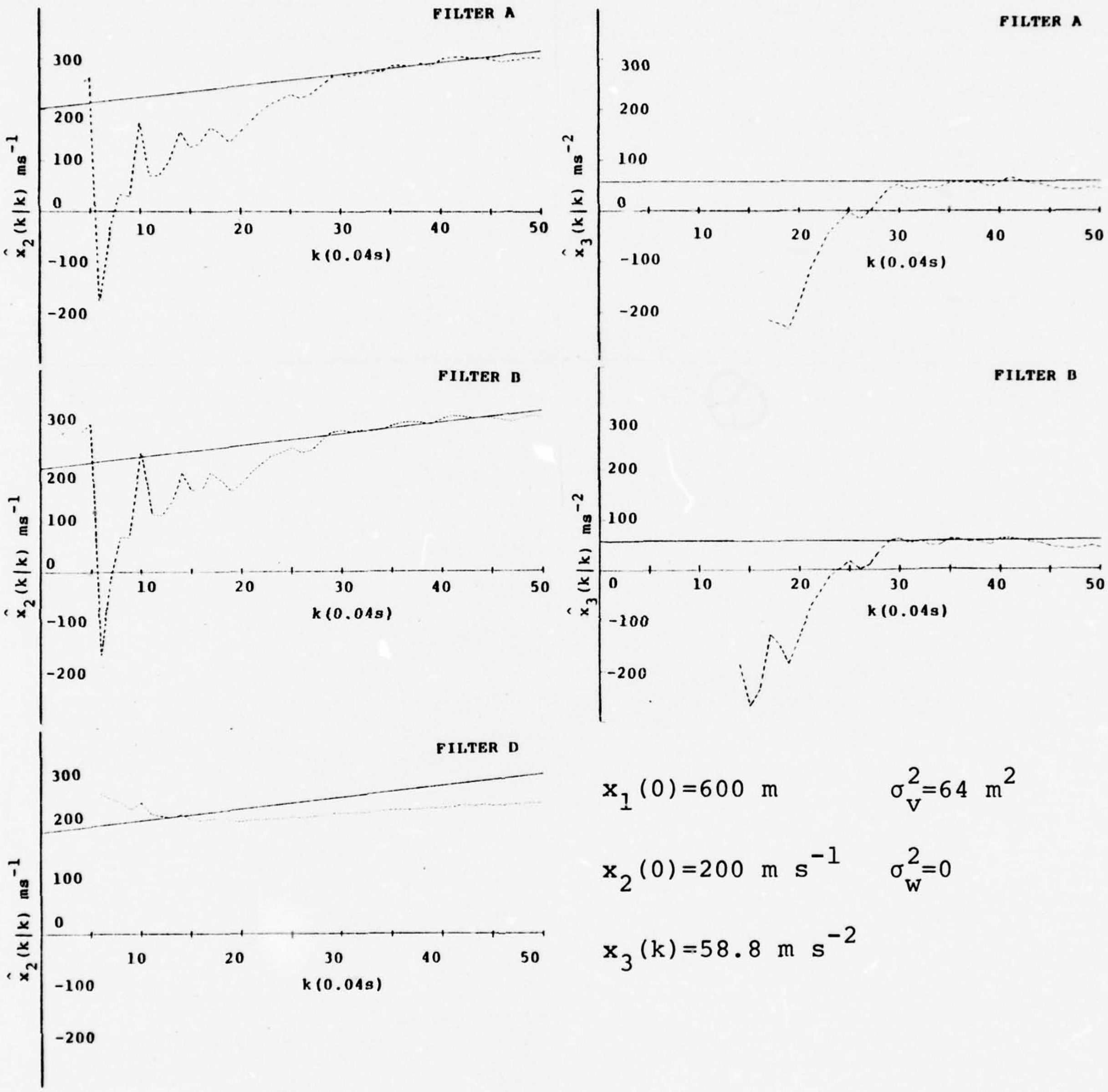


$x_1(0)=600$ m $\qquad \sigma_v^2=64$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=0$

$x_3(k)=9.8$ m s^{-2}

Figure A-11



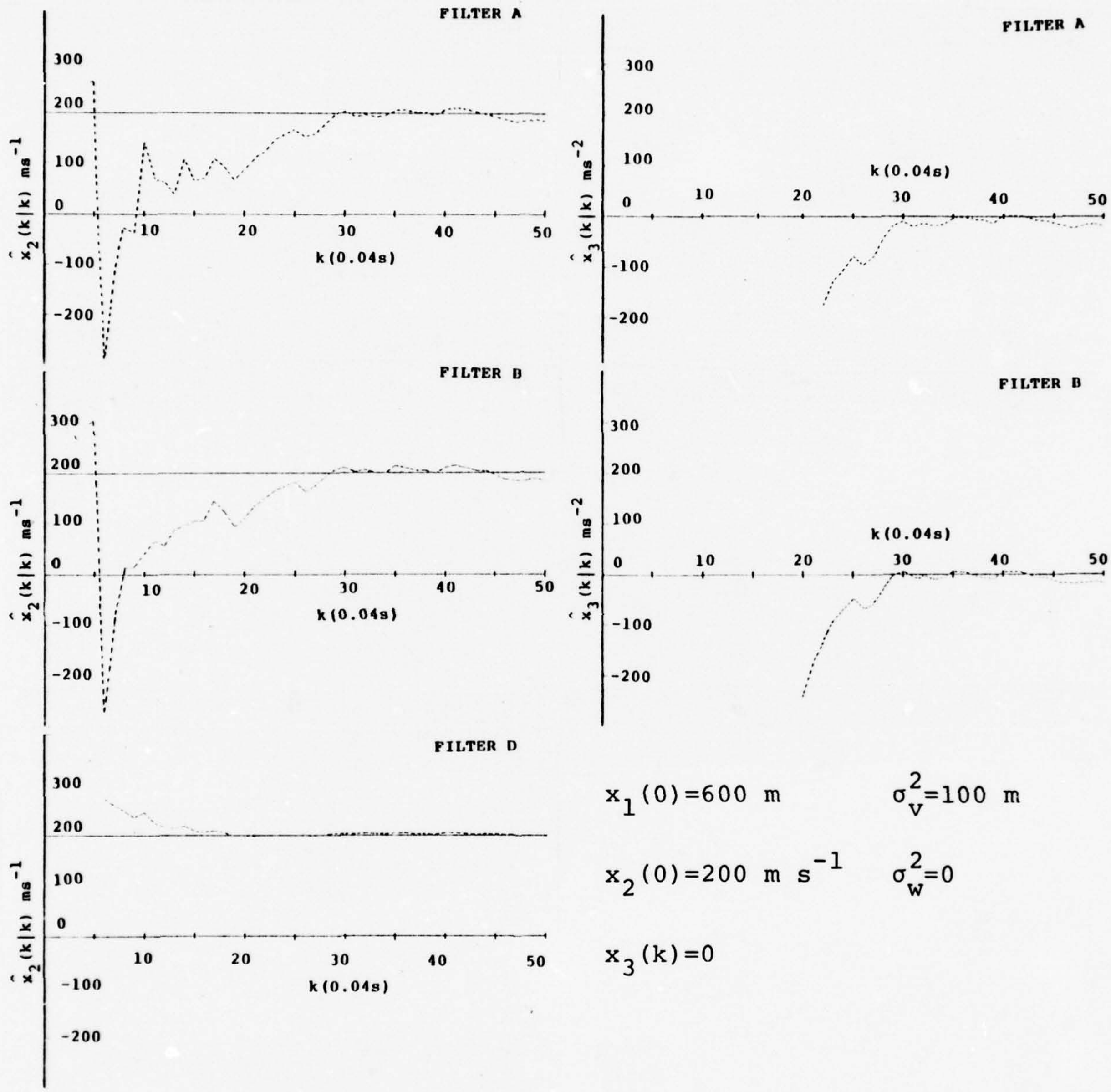
$x_1(0)=600$ m $\qquad \sigma_v^2=64$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=0$

$x_3(k)=58.8$ m s^{-2}

Figure A-12

13



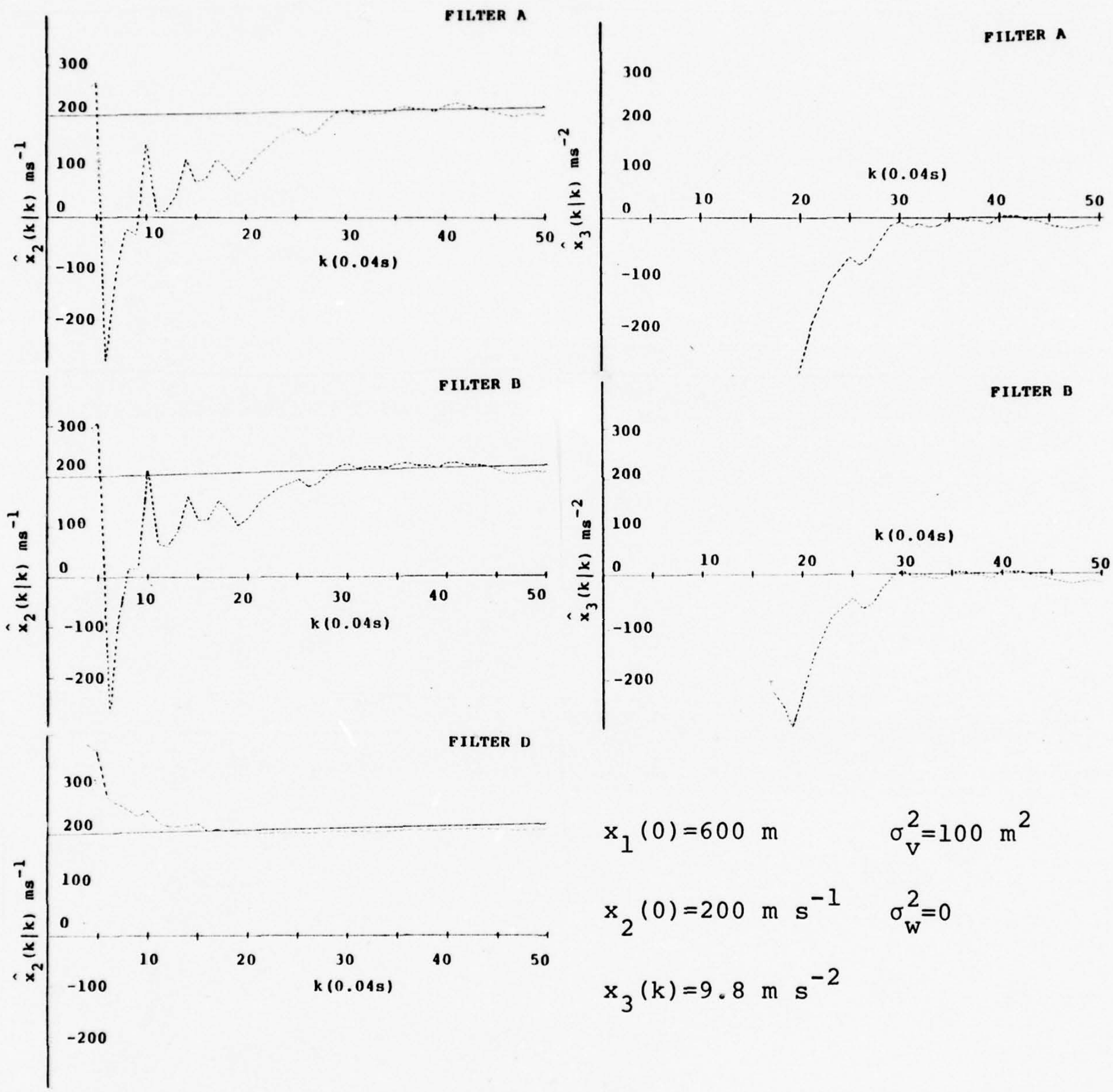
$x_1(0)=600$ m $\quad \sigma_v^2=100$ m

$x_2(0)=200$ m s^{-1} $\quad \sigma_w^2=0$

$x_3(k)=0$

Figure A-13

14

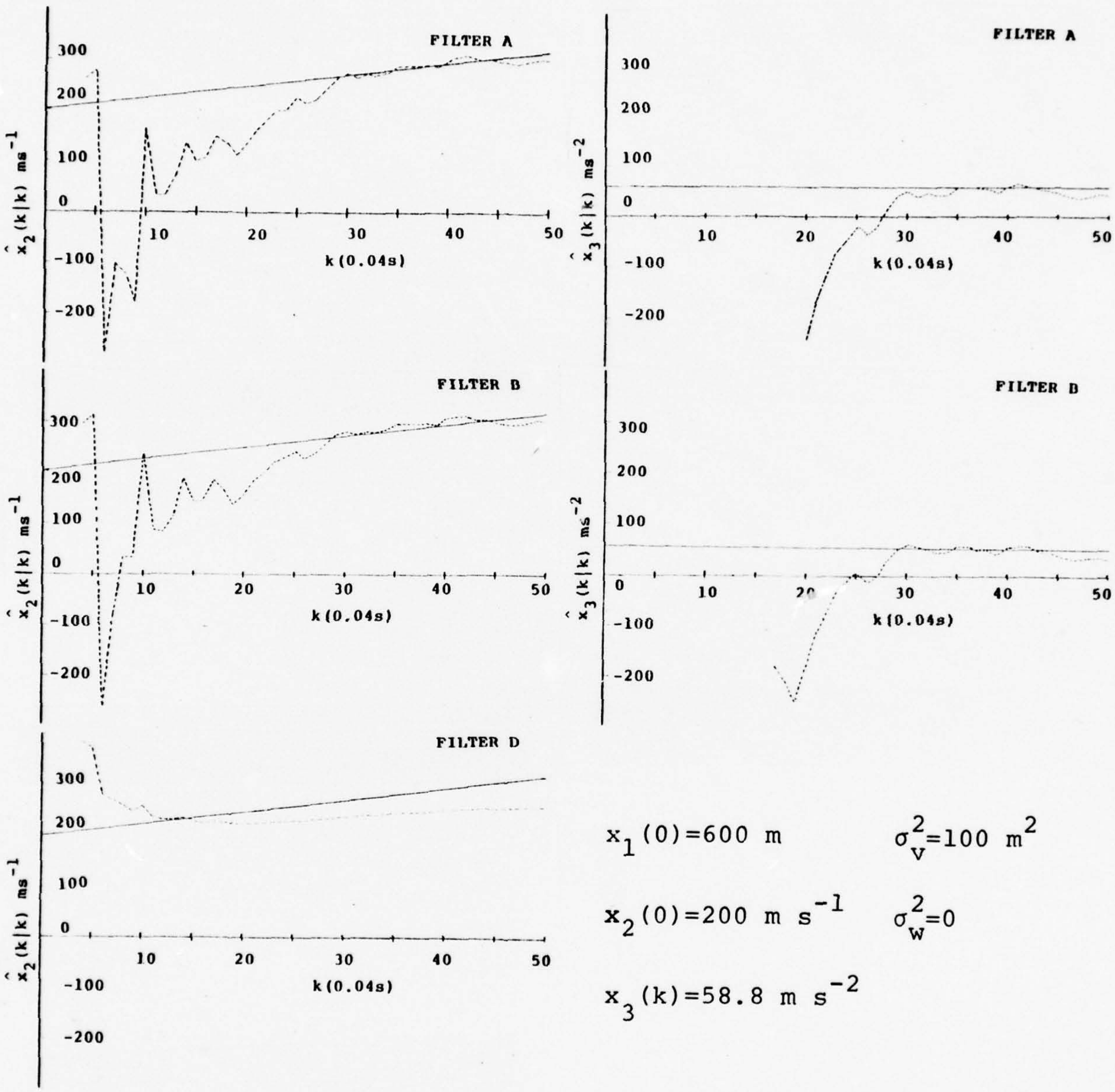


$x_1(0)=600$ m $\quad \sigma_v^2=100$ m^2

$x_2(0)=200$ m s^{-1} $\quad \sigma_w^2=0$

$x_3(k)=9.8$ m s^{-2}

Figure A-14

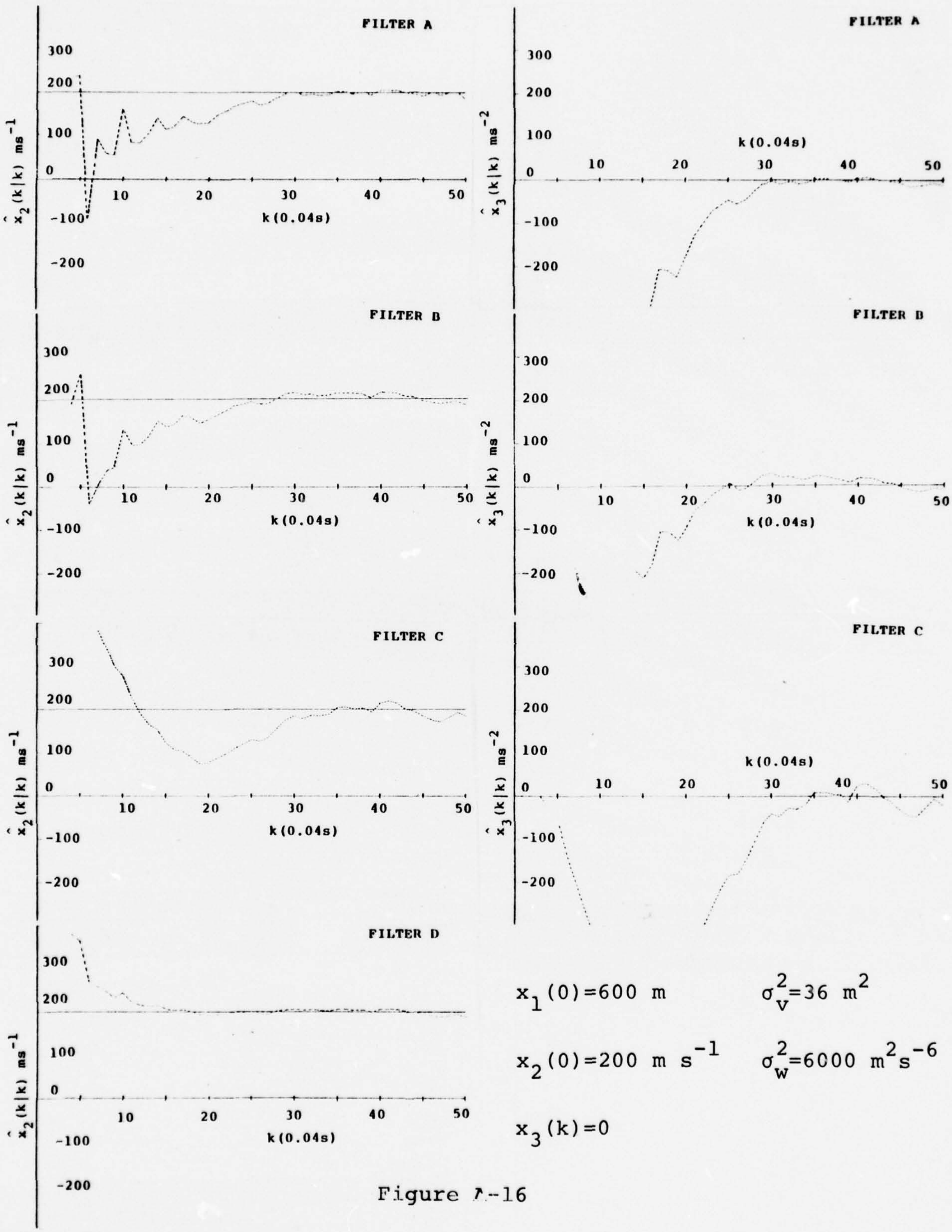


$x_1(0)$=600 m σ_v^2=100 m^2

$x_2(0)$=200 m s^{-1} σ_w^2=0

$x_3(k)$=58.8 m s^{-2}

Figure A-15



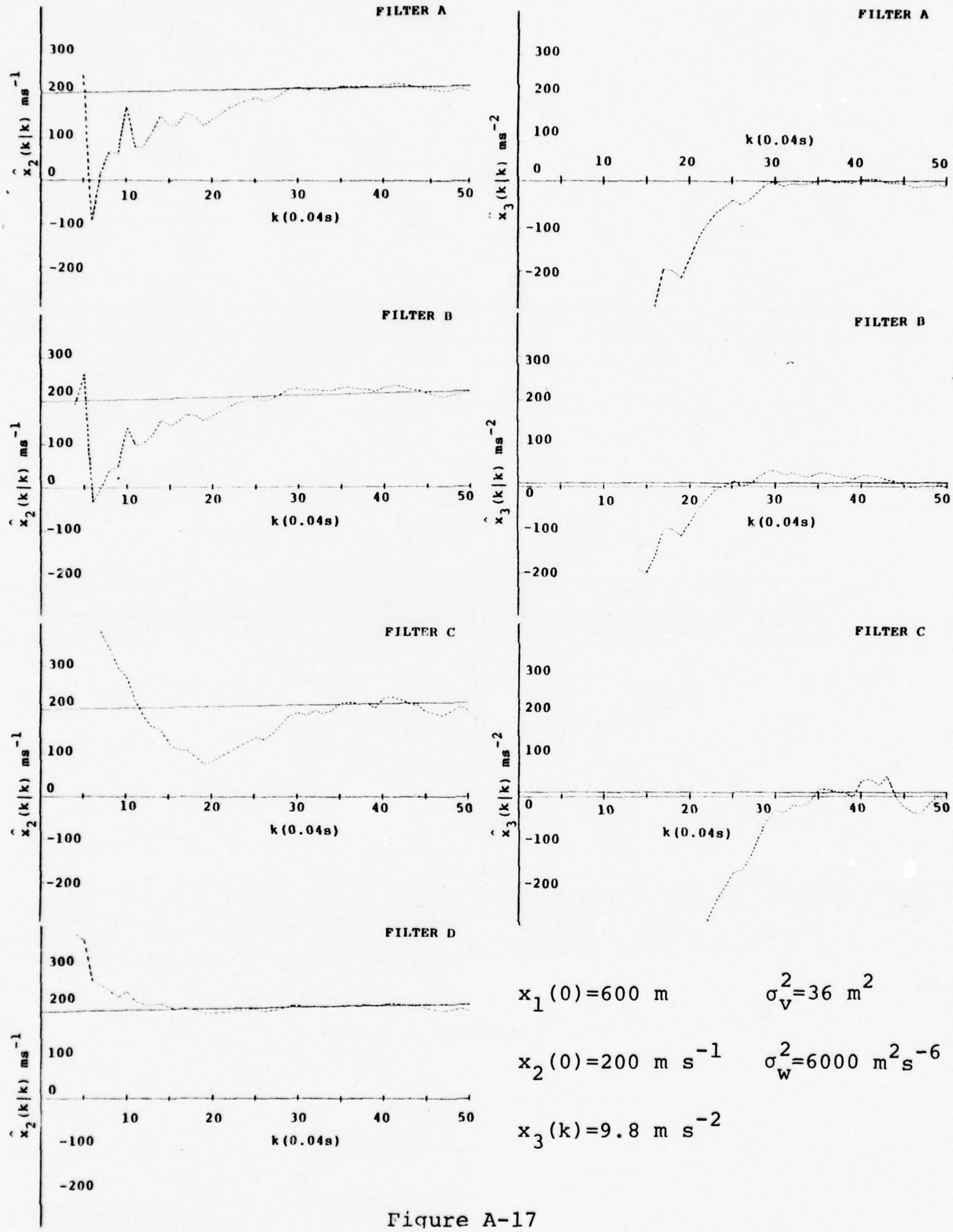
$x_1(0)=600$ m $\sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\sigma_w^2=6000$ m^{2}s^{-6}

$x_3(k)=0$

Figure 7-16

17



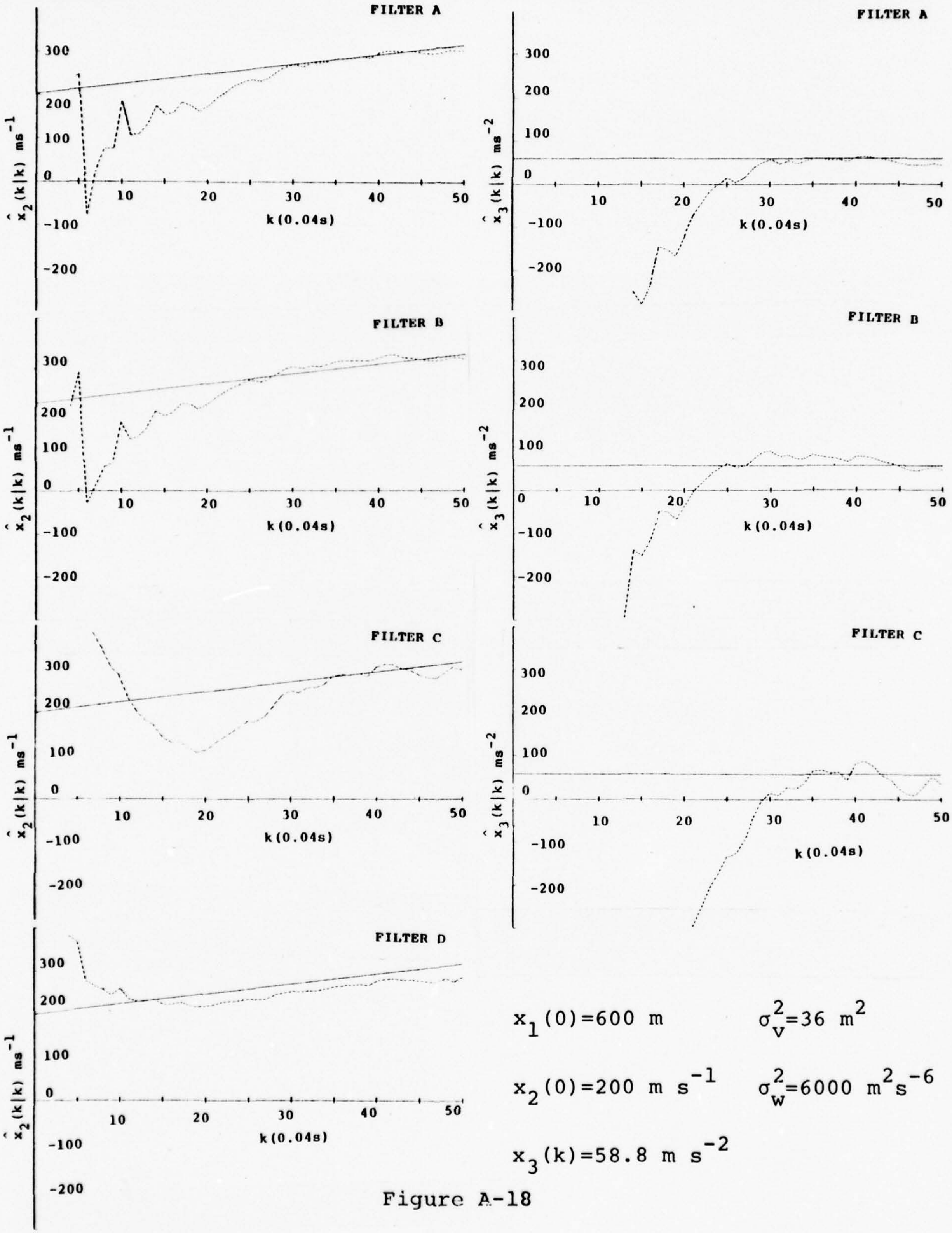
$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=6000$ m^{2}s^{-6}

$x_3(k)=9.8$ m s^{-2}

Figure A-17

18



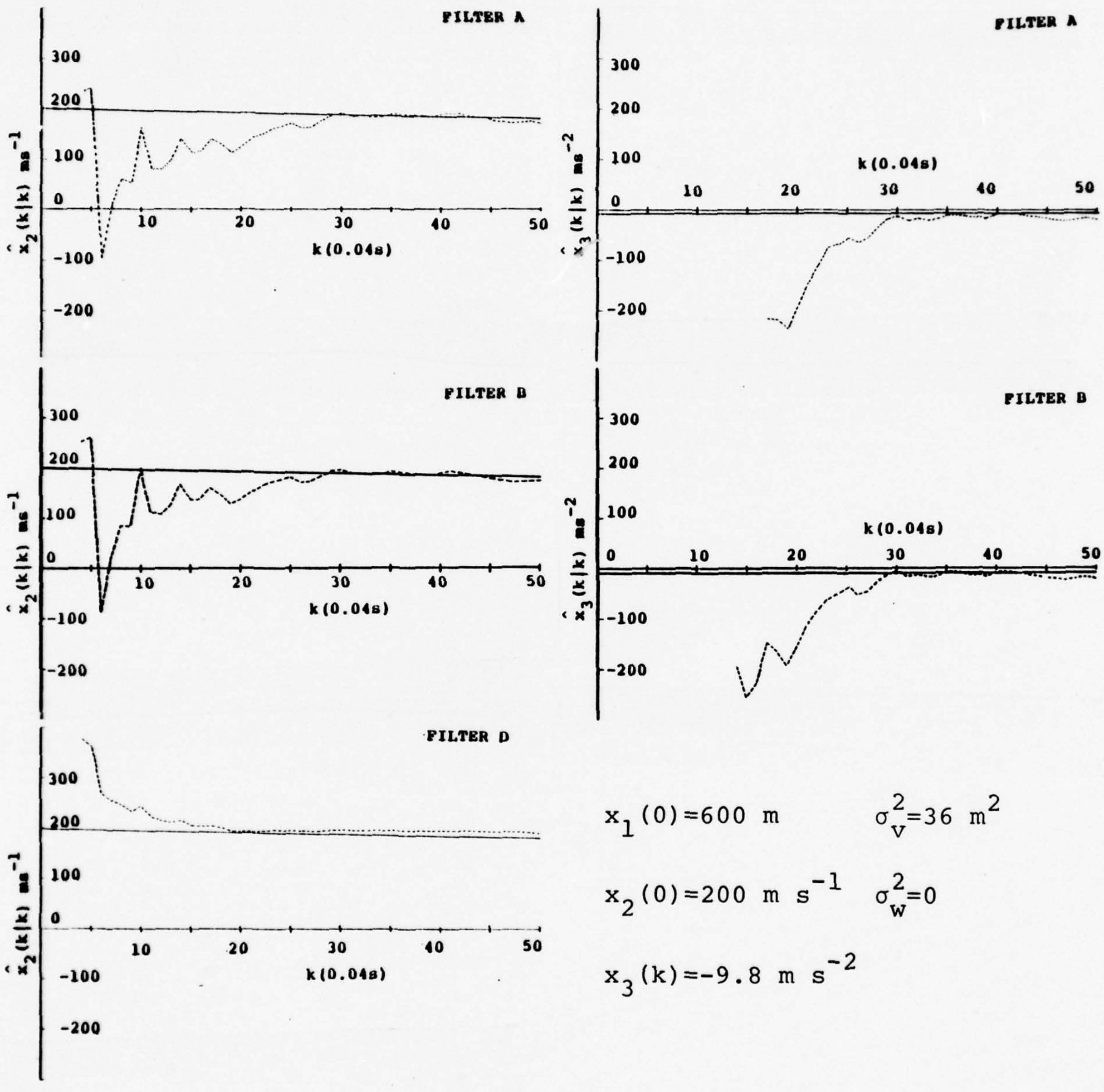
$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=6000$ m^{2}s^{-6}

$x_3(k)=58.8$ m s^{-2}

Figure A-18

19

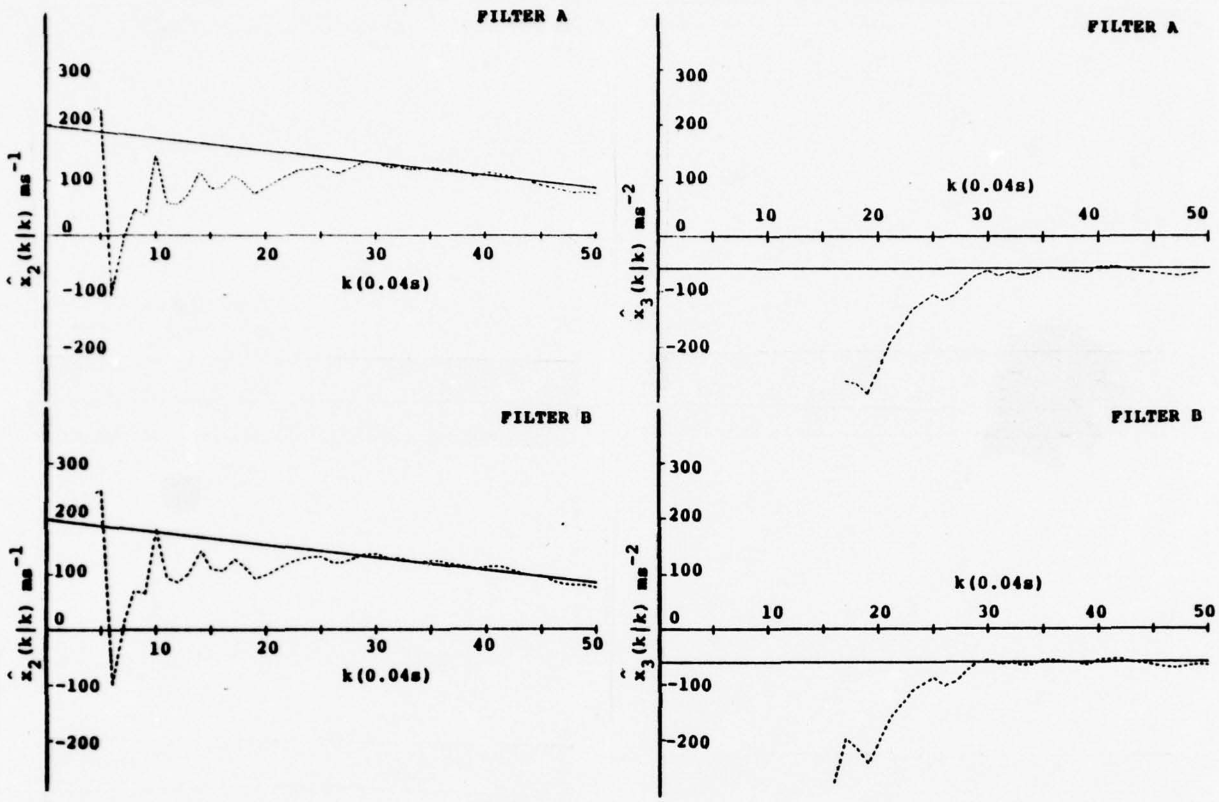


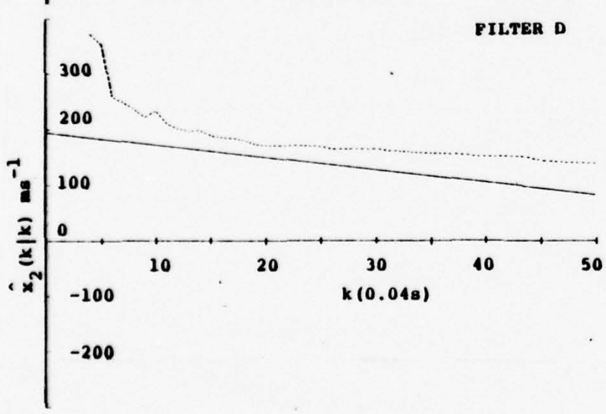
$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=0$

$x_3(k)=-9.8$ m s^{-2}

Figure A-19



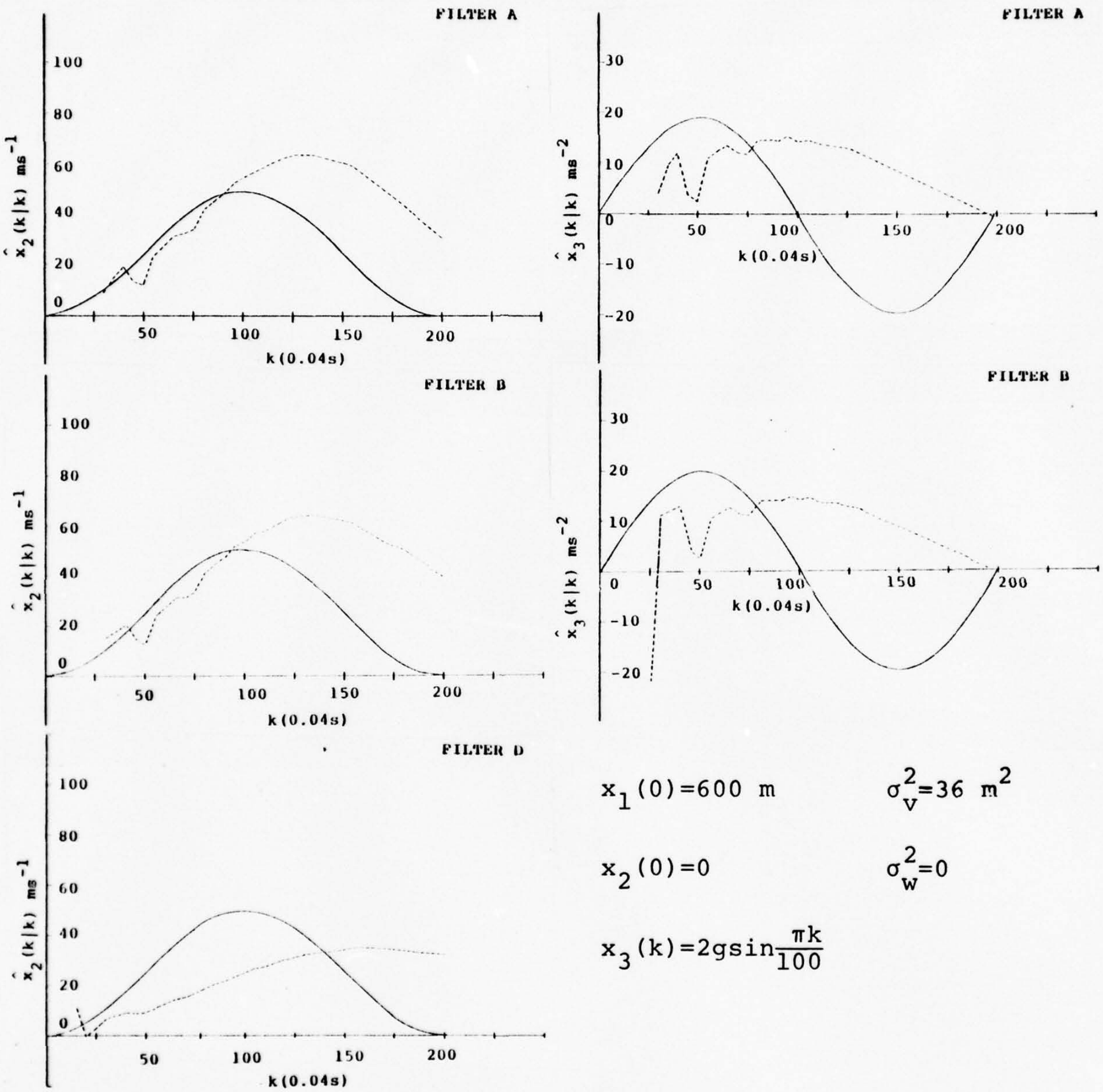


$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=200$ m s^{-1} $\qquad \sigma_w^2=0$

$x_3(k)=58.8$ m s^{-2}

Figure A-20



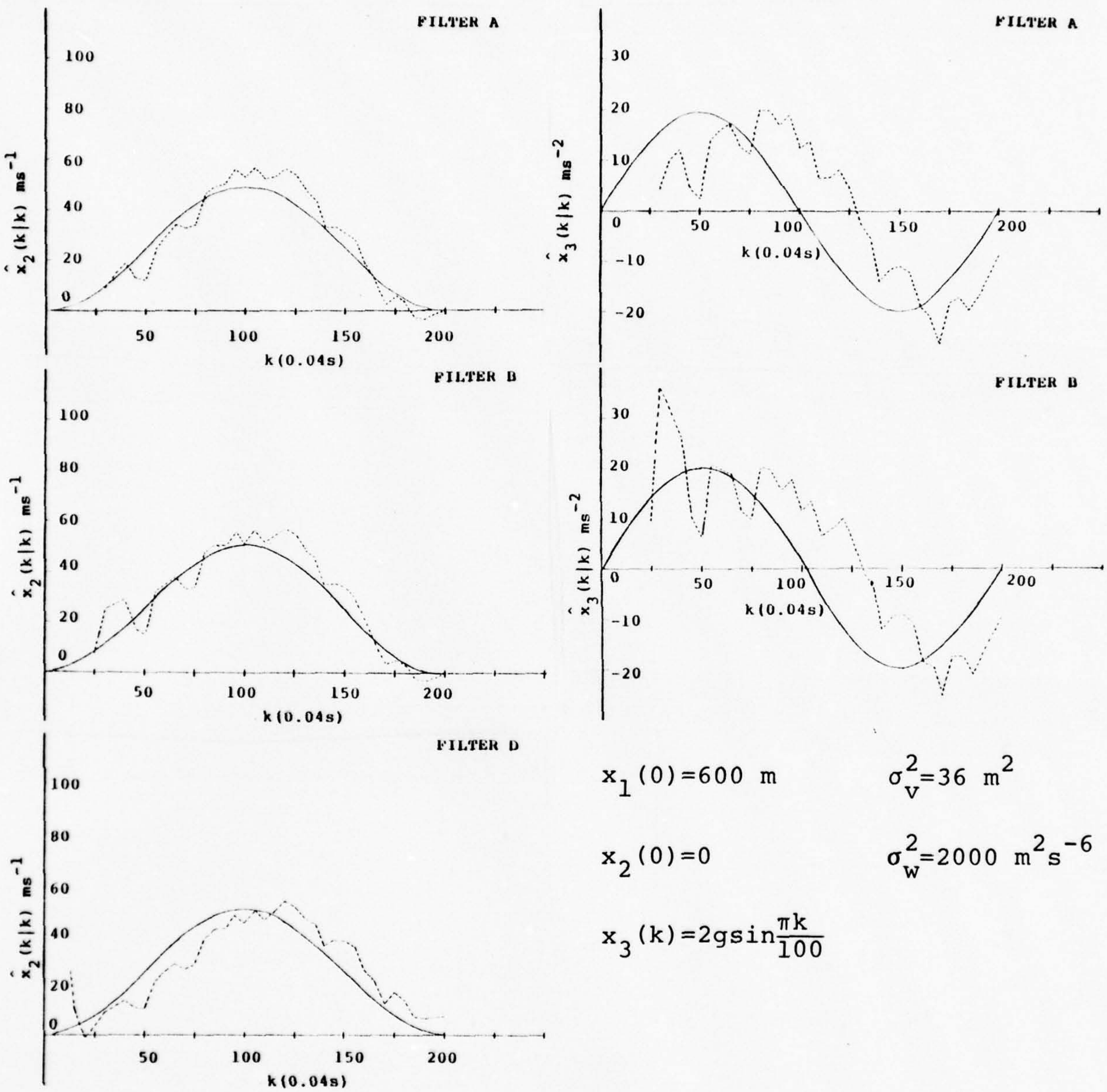
$x_1(0)=600$ m $\quad \sigma_v^2=36$ m^2

$x_2(0)=0 \quad \sigma_w^2=0$

$x_3(k)=2g\sin\frac{\pi k}{100}$

Figure A-21

22

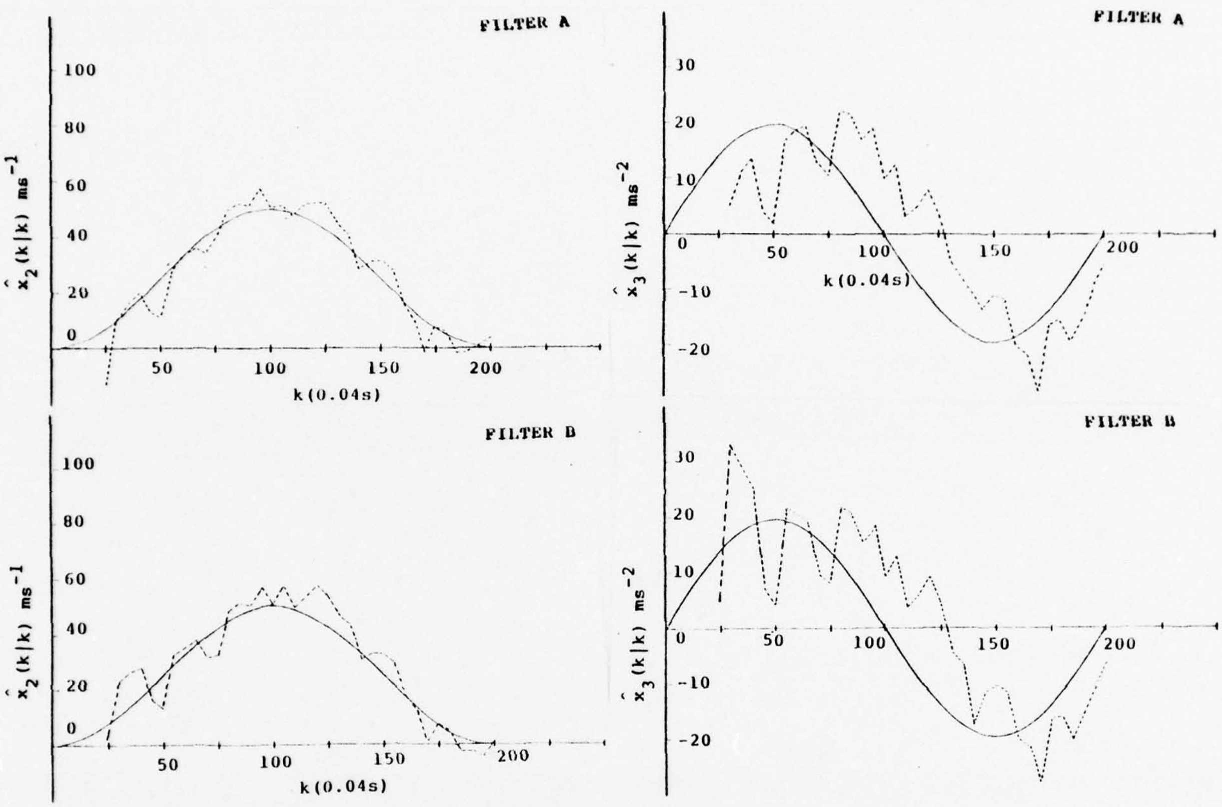


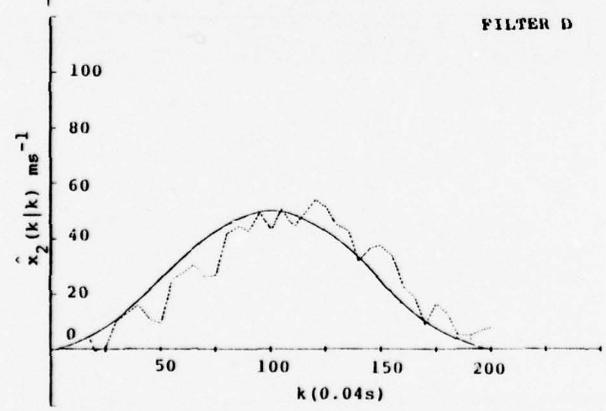
$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=0$ $\qquad \sigma_w^2=2000$ m^{2}s^{-6}

$x_3(k)=2g\sin\frac{\pi k}{100}$

Figure A-22



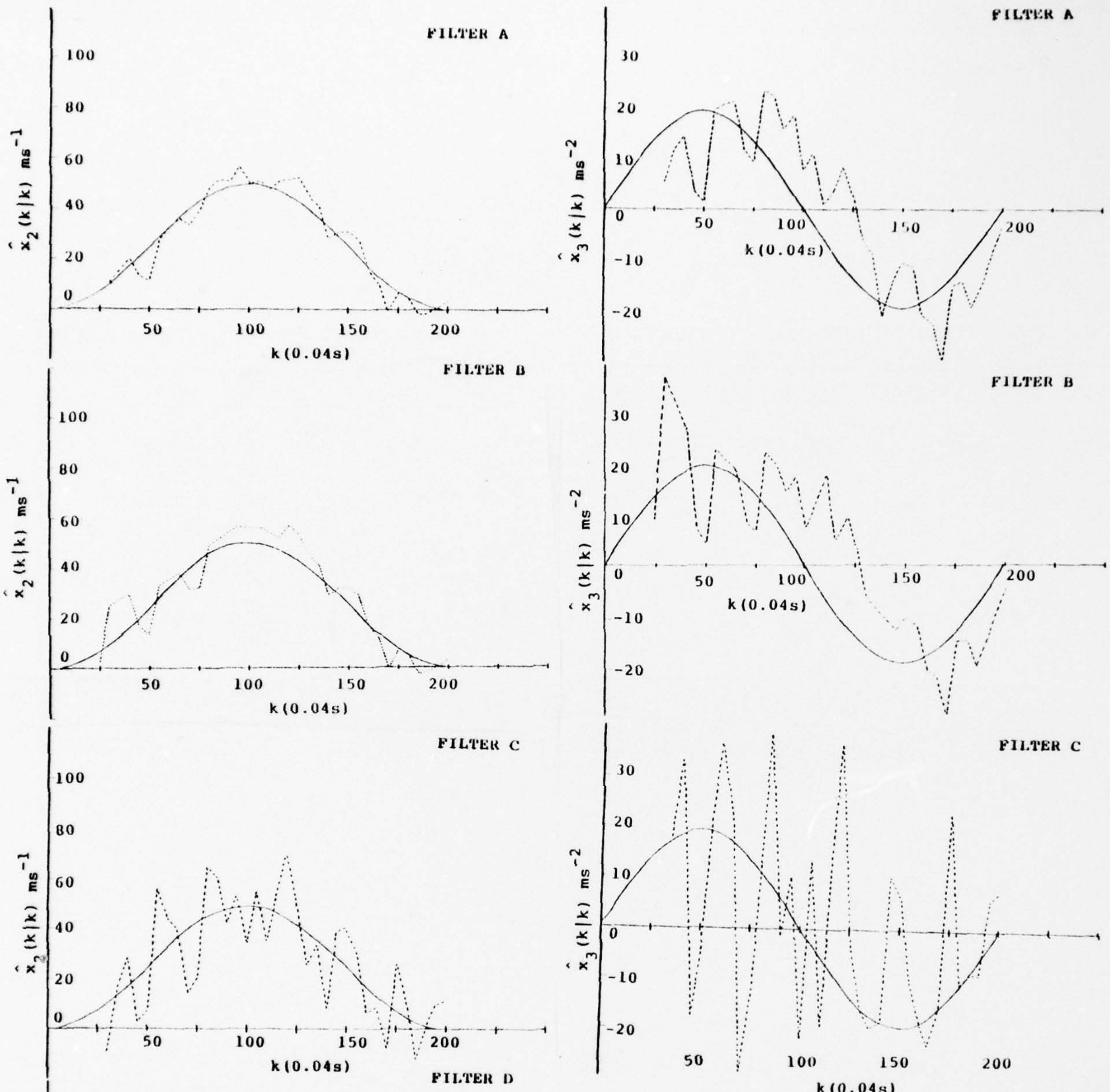


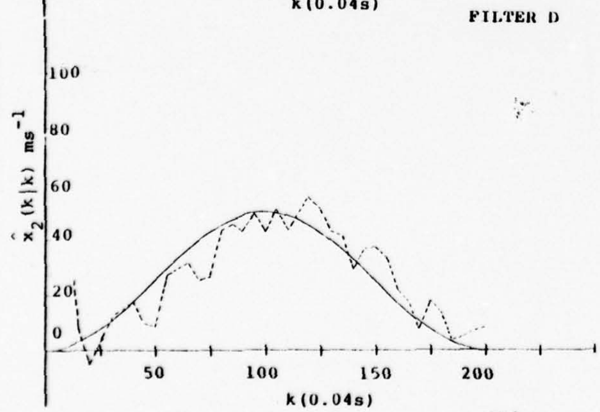
$x_1(0)=600$ m $\qquad \sigma_v^2=36\ m_2$

$x_2(0)=0 \qquad \sigma_w^2=4000\ m^2s^{-6}$

$x_3(k)=2g\sin\frac{\pi k}{100}$

Figure A-23





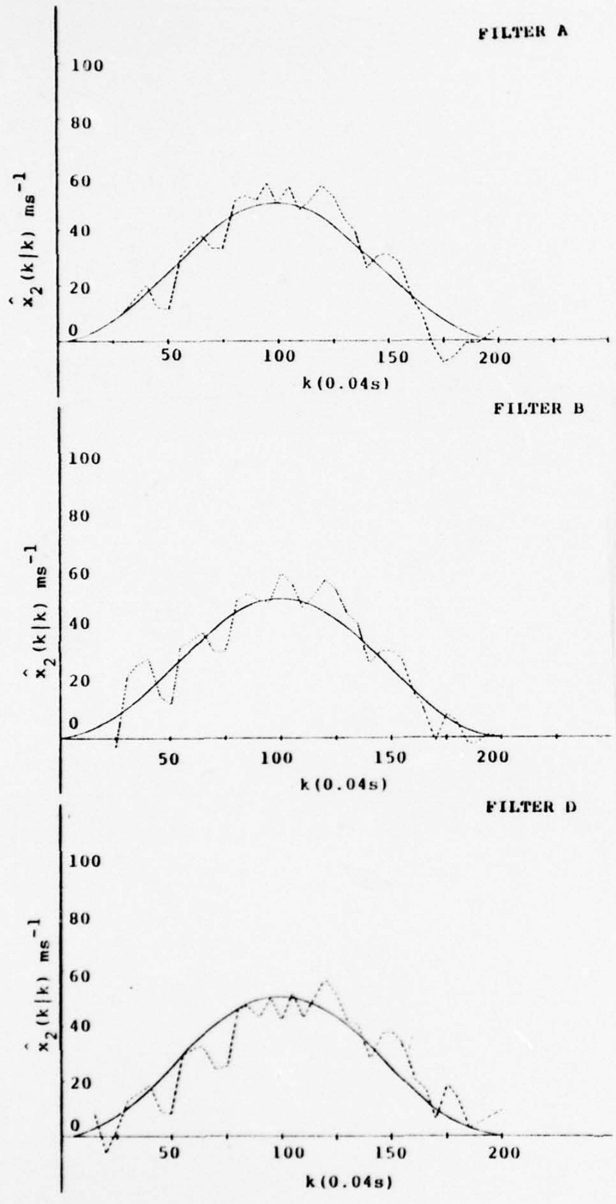
$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

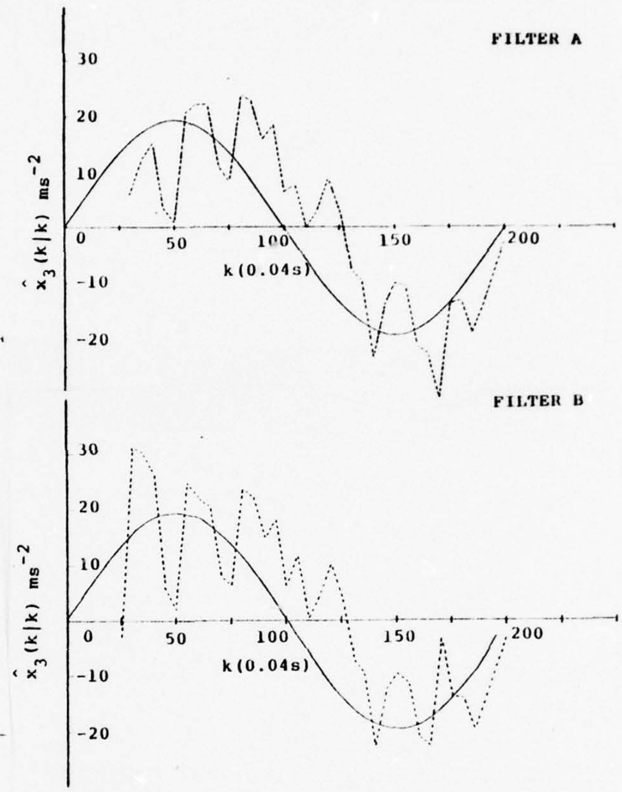
$x_2(0)=0$ $\qquad \sigma_w^2=6000$ m^{2}s^{-6}

$x_3(k)=2g\sin\frac{\pi k}{100}$

Figure A-24

25





$x_1(0)=600$ m

$x_2(0)=0$

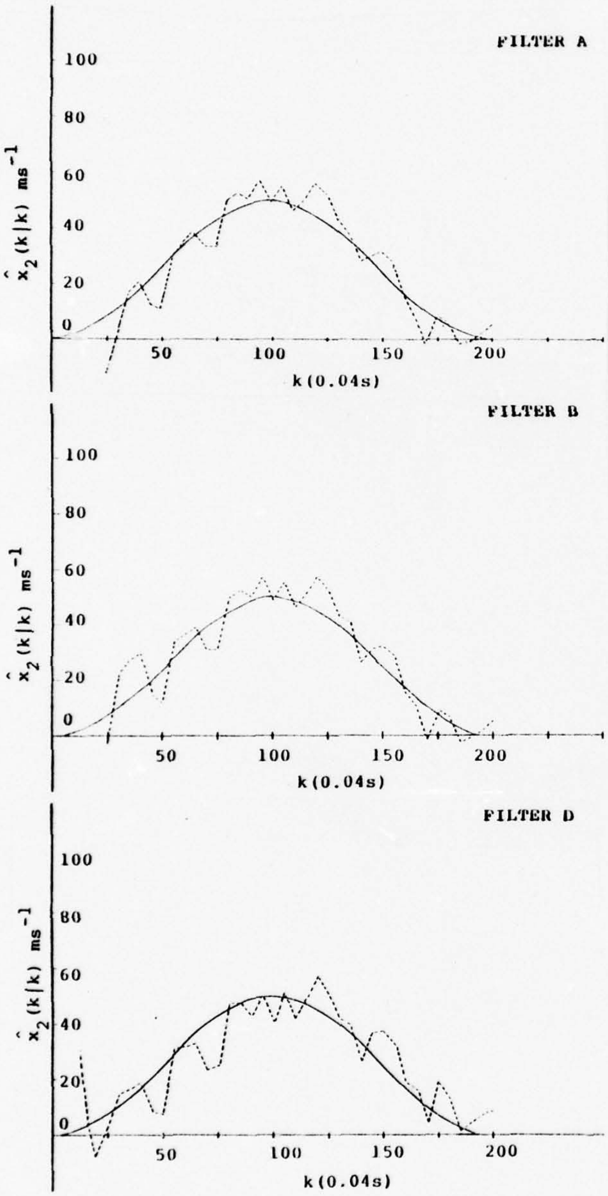
$x_3(k)=2g\sin\frac{\pi k}{100}$

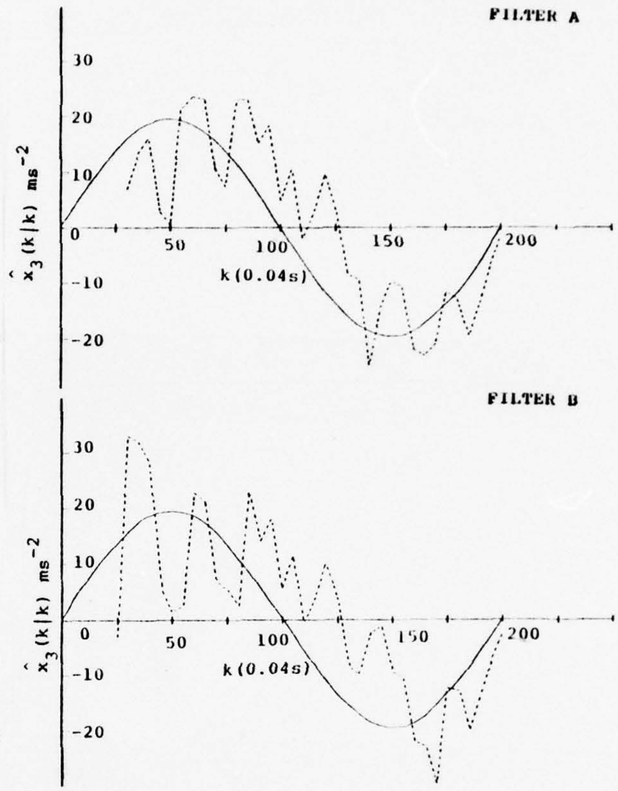
$\sigma_v^2=36$ m^2

$\sigma_w^2=8000$ m^{2}s^{-6}

Figure A-25

26



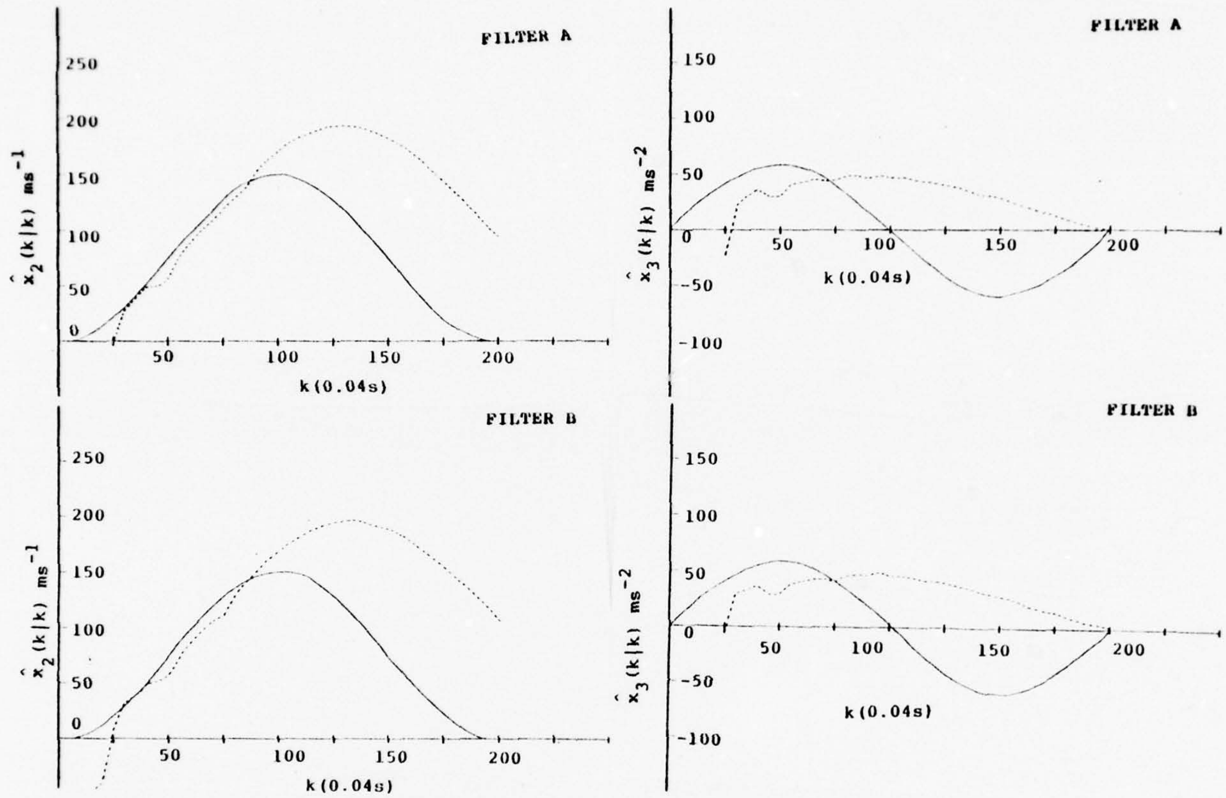


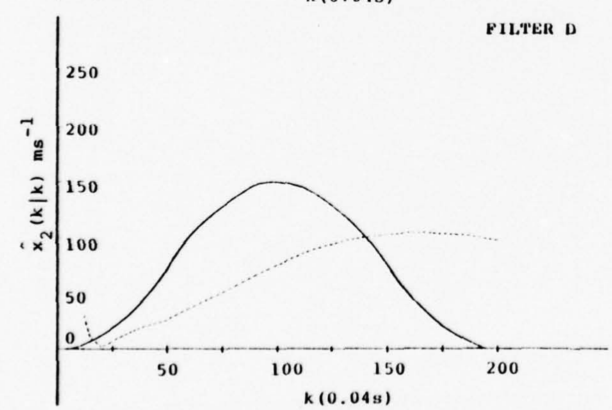
$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=0$ $\qquad \sigma_w^2=10000$ m^{2}s^{-6}

$x_3(k)=2g\sin\frac{\pi k}{100}$

Figure A-26





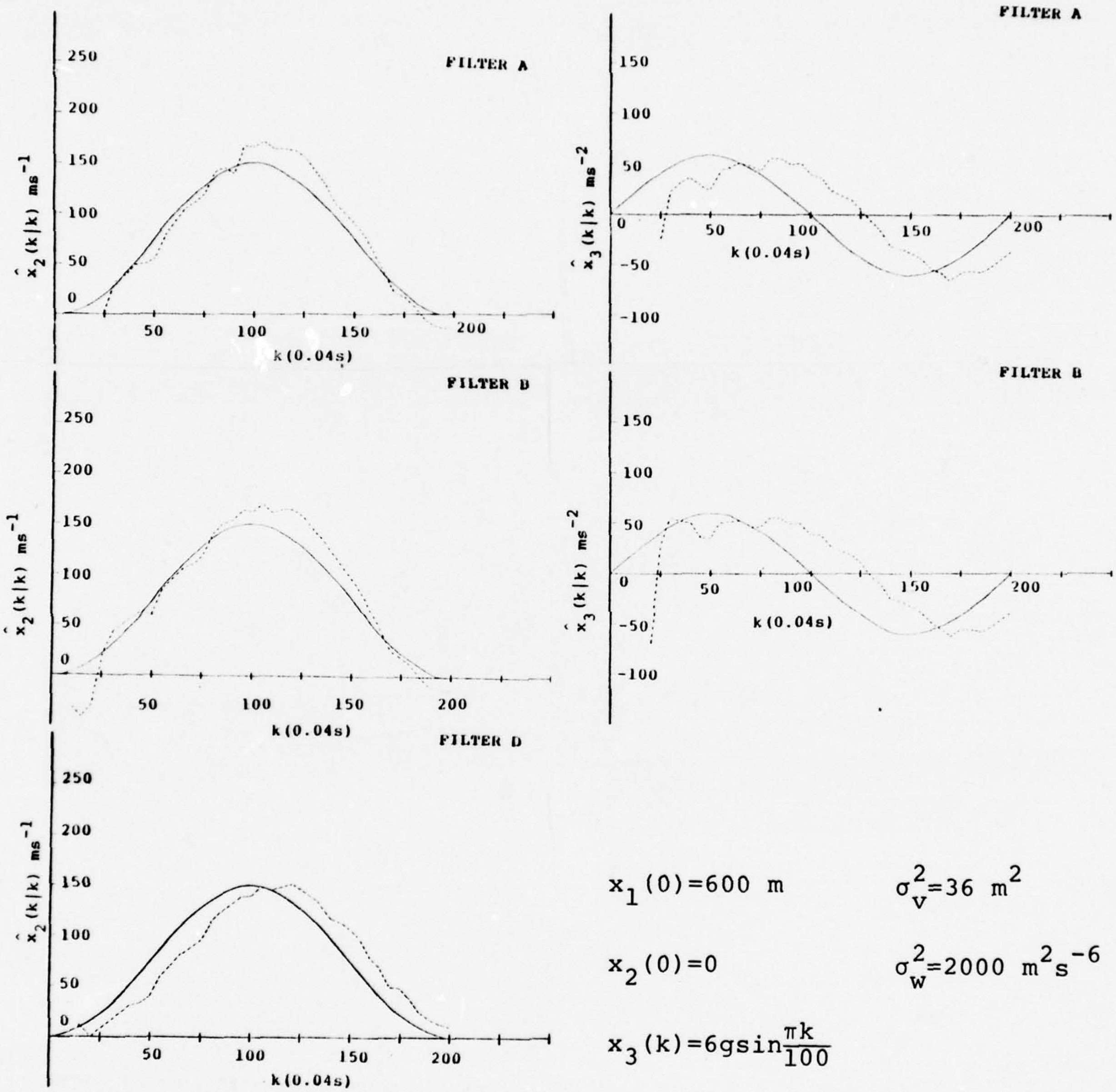
$x_1(0)=600$ m $\qquad \sigma_v^2=36$ m^2

$x_2(0)=0 \qquad \sigma_w^2=0$

$x_3(k)=6g\sin\frac{\pi k}{100}$

Figure A-26

28



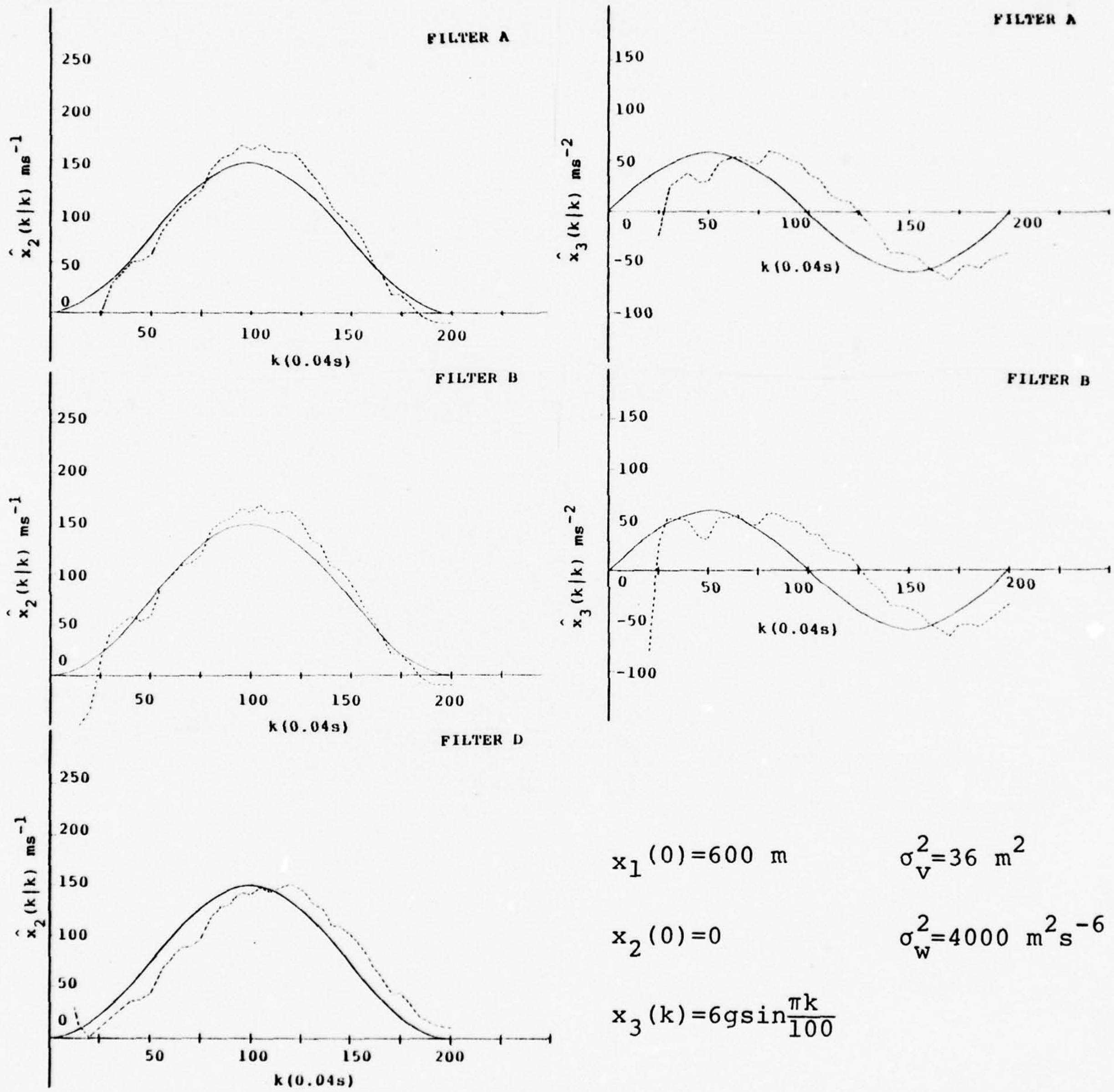
$x_1(0)=600$ m $\sigma_v^2=36$ m^2

$x_2(0)=0$ $\sigma_w^2=2000$ m^{2}s^{-6}

$x_3(k)=6g\sin\frac{\pi k}{100}$

Figure A-28

29



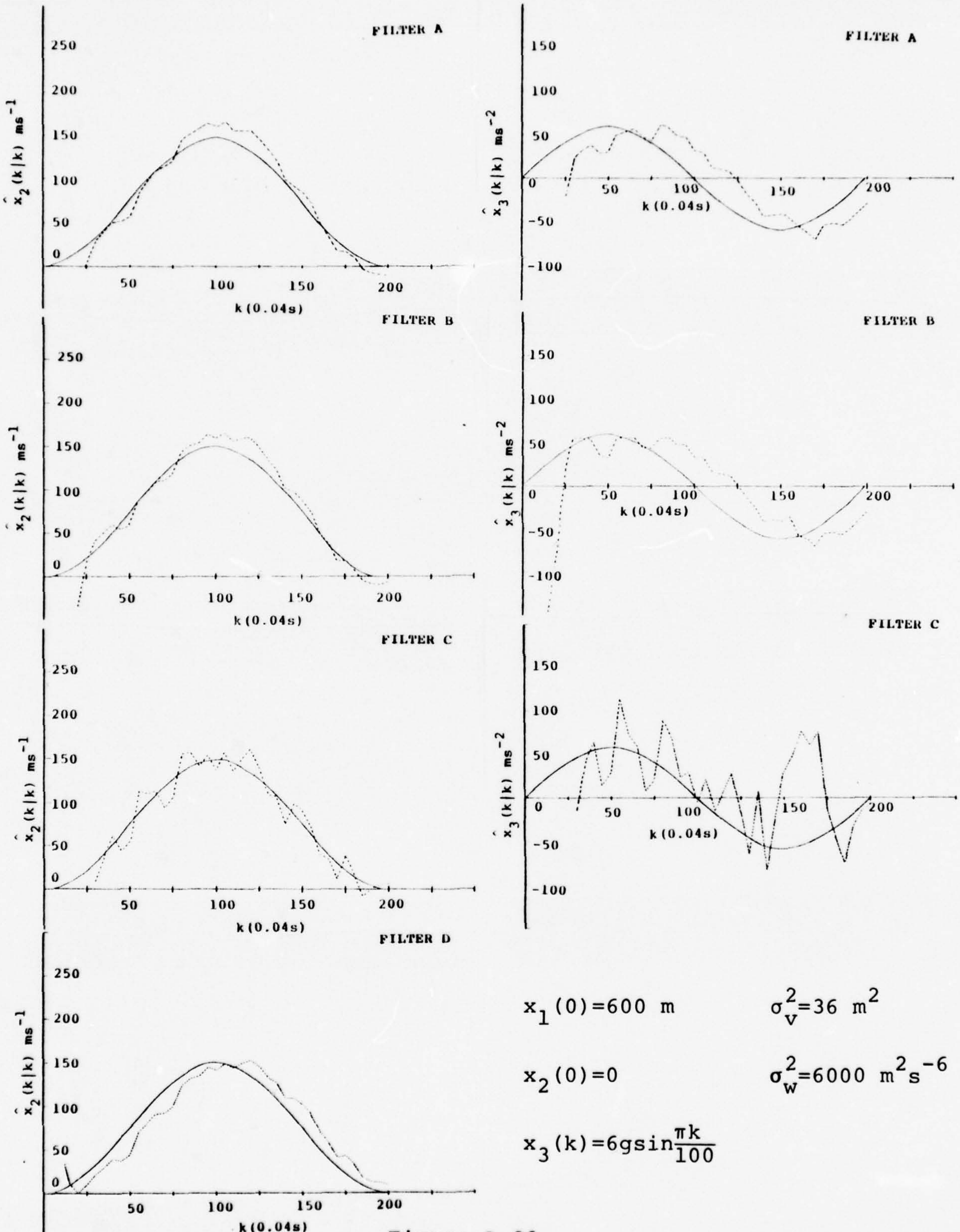
$x_1(0)=600$ m $\qquad \sigma_v^2=36\ m^2$

$x_2(0)=0 \qquad \sigma_w^2=4000\ m^2s^{-6}$

$x_3(k)=6g\sin\frac{\pi k}{100}$

Figure A-29

30



$x_1(0)=600$ m

$\sigma_v^2=36$ m^2

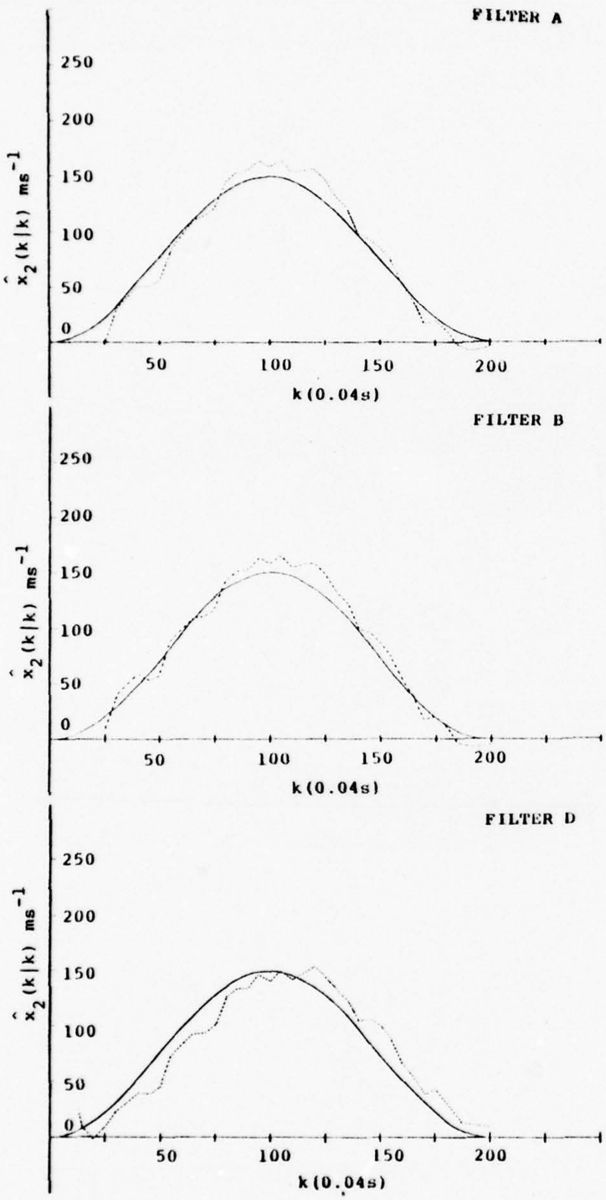
$x_2(0)=0$

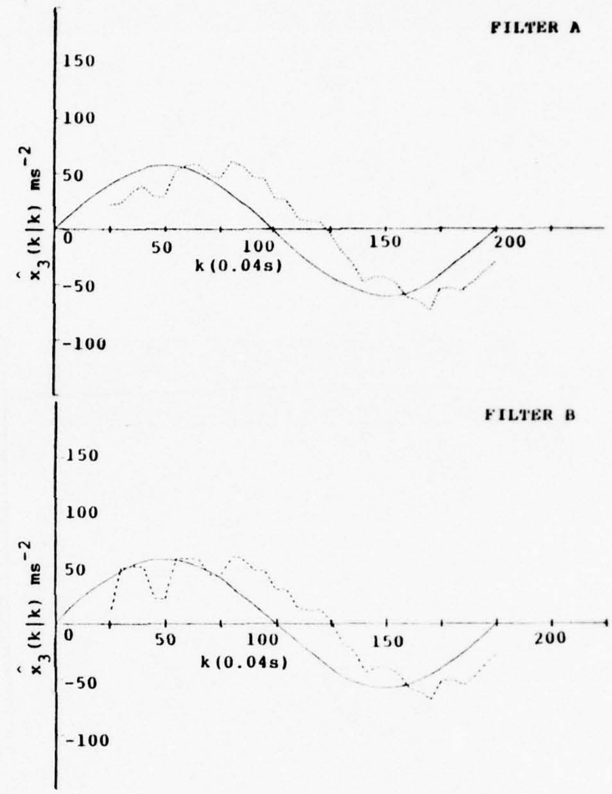
$\sigma_w^2=6000$ m^{2}s^{-6}

$x_3(k)=6g\sin\frac{\pi k}{100}$

Figure A-30

31





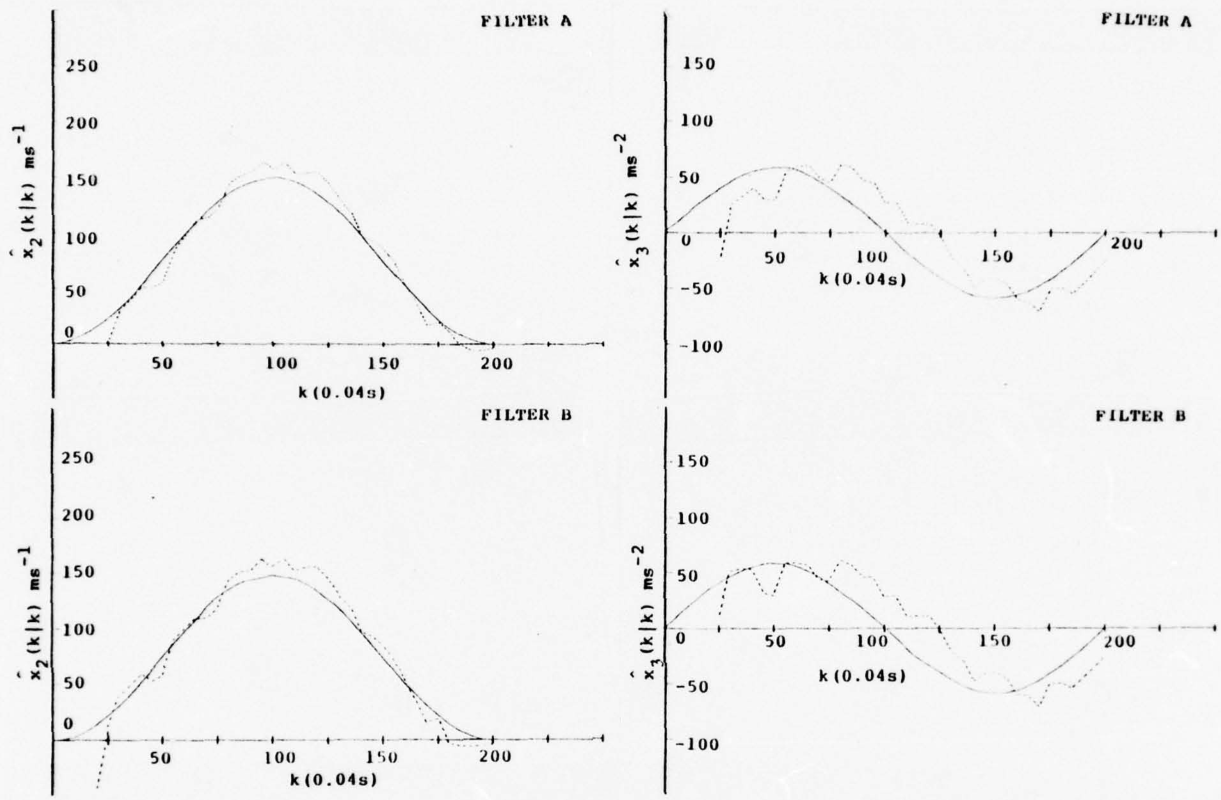
$x_1(0)=600$ m $\qquad \sigma_v^2=36\ m^2$

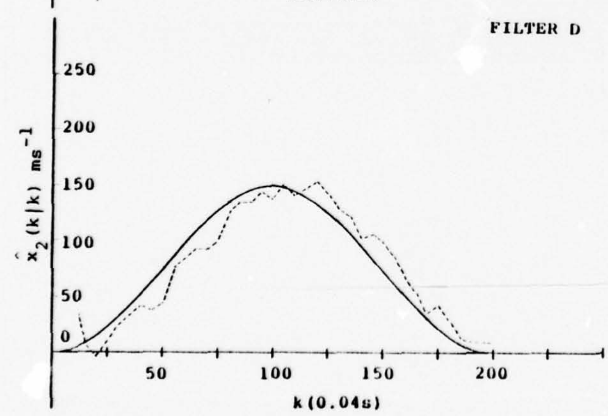
$x_2(0)=0 \qquad \sigma_w^2=8000\ m^2s^{-6}$

$x_3(k)=6g\sin\frac{\pi k}{100}$

Figure A-31

32





$x_1(0)=600$ m $\qquad \sigma_v^2=36\ m^2$

$x_2(0)=0$ $\qquad \sigma_w^2=10000\ m^2s^{-6}$

$x_3(k)=6g\sin\frac{\pi k}{100}$

Figure A-32

APPENDIX B

PROGRAMS FOR KALMAN FILTERS

PART A - <u>CONSTANT ACCELERATION FILTER</u>

PART B - <u>CONSTANT VELOCITY FILTER</u>

*Listing is of the Straight Line Trajectory Model

DESCRIPTION

Computer program 'KALMAN' is a FORTRAN-coded range type Kalman filter. It contains the necessary equations to use one of three models; constant acceleration, sinusoidal acceleration (2G amplitude) and sinusoidal acceleration (6G amplitude). In addition, the Kalman gain may be computed in one of three ways; time varying gain (gain computed each pass as function of the filter parameter'P'), synthetic time varying gain (gain computed each pass as function of time and input coefficients), and constant gain (gain is input).

The program computes statistics which can be used to compare range, velocity and acceleration with their estimated values.

The purpose of the model is two-fold:

1) To provide a means of evaluating several variations of the filter.

2) To enable the user to determine optimum values of some significant filter parameters

The program is not and never was intended to be the end product of the filter study.

METHOD

(1) Calculate 'True' Values of Range, Velocity, and Acceleration (R, $\dot{R}$, $\ddot{R}$) According to One of Three Methods:

- k is an integer which takes on all values from 1 through NPTS (NPTS = duration of run/sampling rate)
- T = sampling rate, sec.

a - Constant Acceleration Model

$$R(k)=R_o+V_o(kT)+\frac{1}{2}A(kT)^2$$

$$\dot{R}(k)=V_o+A(kT)$$

$$\ddot{R}(k)=A$$

b - Sinusoidal Acceleration, 2G Amplitude

$$R(k)=R_o+24.955[(kT)-1.27324\mathrm{SIN}(.031416\ k)]$$

$$\dot{R}(k)=24.955[1.0-\mathrm{COS}(.031416\ k)]$$

$$\ddot{R}(k)=19.60\mathrm{SIN}(.031416\ k)$$

c - Sinusoidal Acceleration, 6G Amplitude

$$R(k)=R_o+2.99466[k-31.831\mathrm{SIN}(o.31416\ k)]$$

$$\dot{R}(k)=74.8665[1.0-\mathrm{COS}(.031416\ k)]$$

$$\ddot{R}(k)=58.8\mathrm{SIN}(.031416\ k)$$

(2) Generation of Gaussian Noise:

Generate 12 random numbers each having a uniform distribution between 0 and 1 (represented by RN_i)

Then the noise, $n(k)= \sigma_v[\sum_{i=1}^{12} RN_i)-6.0]$ is a Quasi-Gaussian random variable with zero mean and standard deviation=σ_v

(3) The 'Measured' Value of Range is $Z(k)=R(k)+n(k)$

(4) X_1, X_2, X_3 = Estimates of R, $\dot{R}$, $\ddot{R}$

To initialize: $X_1 = Z(3)$

$X_2 = 12.5[3.0Z(3) - 4.0Z(2) + (1)]$

$X_3 = 0.0$

(5) Filter Equations

a) $P(k|k-1) = (\phi)\ P(k-1|k-1)\ (\phi T) + S(k)$

$$\phi = \text{Constant Matrix} = \begin{vmatrix} 1 & T & T^2/2.0 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{vmatrix}$$

ϕ^T = Transpose of ϕ

$$S(k) = \text{Maneuver Noise Term} = \sigma_w^2 T^2 / \sigma_v^2 \begin{vmatrix} T^4 & 3T^3 & 6T^2 \\ 3T^3 & 9T^2 & 18T \\ 6T^2 & 18T & 36.0 \end{vmatrix}$$

$$P \text{ is initialized as} \cdots \sigma_v^2 \begin{vmatrix} 1 & (3/2T) & (1/T^2) \\ (3/2T) & (13/2T^2) & (6/T^3) \\ (1/T^2) & (6/T^3) & (6/T^4) + \sigma_w^2 \end{vmatrix}$$

b) Gain (Time Varying Gain)

$$\underline{K}(k) = \frac{1.0}{P_{1,1}(k|k-1) + \sigma_v^2} \begin{vmatrix} P_{1,1}(k|k-1) \\ P_{2,1}(k|k-1) \\ P_{3,1}(k|k-1) \end{vmatrix}$$

c) Estimates:

$$\underline{\hat{X}}(k|k) = (\phi)\ \underline{\hat{X}}(k|k-1) + \underline{K}(k)\,[Z(k) - (H)\,(\phi)\,\underline{\hat{X}}(k|k-1)]$$

where (H) = Row Matrix = (1,0,0)

d) $P(k|k) = [\underline{T} - \underline{K}(k)\,(H)]\ \ P(k|k-1)$

$$\text{where } \underline{T} = \text{Identity Matrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

* Note - For synthetic time varying gain and constant gain equations (a) and (d) are By-Passed for the constant gain case, $\underline{K}_i$=input values for synthetic time varying gain, ···

$$\underline{K}_1 = \frac{C1}{k+C2} + C3; \quad \underline{K}_2 = \frac{C4}{k^2+C5} + C6; \quad \underline{K}_3 = \frac{C7}{k^3+C8} + C9$$

where C1, C2,---,C9 are input

(6) Statistics - The Following RMS Deviations are Calculated:

$$(\Delta X_1)\,RMS = \sqrt{\frac{1.0}{(N2-N1+1)} \sum_{k=N1}^{N2} [R(k)=\hat{X}_1(k|k)]^2}$$

$$(\Delta X_2)\,RMS = \sqrt{\frac{1.0}{(N2-N1+1)} \sum_{k=N1}^{N2} [\dot{R}(k)-\hat{X}_2(k|k)]^2}$$

$$(\Delta X_3)\,RMS = \sqrt{\frac{1.0}{(N2-N1+1)} \sum_{k=N1}^{N2} [\ddot{R}(k)-\hat{X}_3(k|k)]^2}$$

N1, N2 are allowed to take on 3 sets of values, one of which is always N1=4; N2=NPTS. The other two sets are input.

PREPARATION OF INPUT CARDS

Note ··· F.PT. = Floating Point, I=Integer

CARD TYPE #1 (Mandatory)

Var.	MNEMONIC	COLS.	TYPE	DEFINITION
	IMEAS	5	I	Model Indicator: 0 = Constant Acceleration, R, $\dot{R}$, $\ddot{R}$ computed 1 = Constant Acceleration, R Measurements Input 2 = Sinusoidal Acceleration, 2g Amplitude 3 = Sinusoidal Acceleration, 6g Amplitude
	IGAIN	10	I	Gain Indicator: 0 = Time Varying Gain 1 = Constant Gain 2 = Synthetic Time Varying Gain
	INTIA	11-15	I	Statistics Interval #1, First Point
	INTIB	16-20	I	Statistics Interval #1, Last Point
	INT2A	21-25	I	Statistics Interval #2, First Point
	INT2B	26-30	I	Statistics Interval #2, Last Point
T_R	TRATE	31-40	F.PT.	Sampling Rate, Sec.
	SECS	41-50	F.PT.	Total Time of Run, Sec. (SECS/TRATE = 250 Maximum)

CARD TYPE #2 (Mandatory)

Var.	MNEMONIC	COLS.	TYPE	DEFINITION
σ_v^2	SVSQ	1-10	F.PT.	Variance, Measurement Noise
σ_w^2	SWSQ	11-20	F.PT.	Variance, Manuever Noise
r_o	RO	21-30	F.PT.	Initial Range, Meters (Not required if IMEAS=1)
v_o	VO	31-40	F.PT.	Initial Velocity, M/SEC. (Not required if IMEAS=2,3)

a	A	41-50	F.PT.	Acceleration, M/SEC^2 (Not required if IMEAS=2,3)
K_1	K_1	51-60	F.PT.	Kalman Gain (Required for IGAIN=1 Only)
K_2	K_2	61-70	F.PT.	Kalman Gain (Required for IGAIN=1 Only)
K_3	K_3	71-80	F.PT.	Kalman Gain (Required for IGAIN=1 Only)

<u>CARD TYPE #3</u> (This Card Required for Synthetic Time Varying Gain Only --- IGAIN=2)

Var.	MNEMONIC	COLS.	TYPE	DEFINITION
C_1	C1	1-10	F.PT.	Coefficients---See Equations On Page 3
C_2	C2	11-20	F.PT.	
C_3	C3	21-30	F.PT.	
C_4	C4	31-40	F.PT.	
C_5	C5	41-50	F.PT.	
C_6	C6	51-60	F.PT.	
C_7	C7	61-70	F.PT.	
C_8	C8	71-80	F.PT.	
C_9	C9	1-10, 2^{nd} CD.	F.PT.	

<u>CARD TYPE #4</u> (This Card Required for IMEAS=1 Only)

Enter successive values of range measurements 8 per card, 8F10.1 Format. Use number of cards required, but number of values must not exceed 250.

OUTPUT

The program output consists of three (3) parts:

1) Headings (type of run, values of input parameters, etc.). Plus the values of R, $\dot{R}$, $\ddot{R}$, $\hat{X}_1$, $\hat{X}_2$, $\hat{X}_3$, K_1, K_2, K_3 at each point.

2) The measurement and noise at each point.

3) Statistics computed at three intervals, two of which are input and the last of which is for the range ($4 \rightarrow$ NPTS).

```
      PROGRAM KALMAN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C*KALMAN FILTER---TIME VARYING,SYNTHETIC TIME VARYING,OR CONSTANT GAIN
C*---           CONST ACCELERATION,,RADIAL MANUEVER,OR INPUT TRAJECTORY
C***************************************************************
C*INPUT:
C*    IMEAS=0,CONSTANT ACCELERATION
C*         =1,TRAJECTORY INPUT
C*         =2,2G TURN
C*         =3,5G TURN
C*    IGAIN=0,TIME VARYING GAIN
C*         =1,CONSTANT GAIN
C*         =2,SYNTHETIC TIME VARYING GAIN
C*    INT1A=1ST PT,1ST INTERVAL,STATISTICS
C*    INT1B=2ND PT,1ST INTERVAL,STATISTICS
C*    INT2A=1ST PT,2ND INTERVAL,STATISTICS
C*    INT2B=2ND PT,2ND INTERVAL,STATISTICS
C*    TRATE=SAMPLING RATE,SECONDS
C*    SECS=TOTAL NUMBER OF SECONDS    (SECS/TRATE=250 MAX)
C*    SVSQ=VARIANCE,MEASUREMENT NOISE
C*    SWSQ=VARIANCE,MANUEVER NOISE
C*    R0=INITIAL RANGE,METERS
C*    V0=INITIAL VELOCITY,M/SEC
C*    A=ACCELERATION,M/SEC**2
C*    K1,K2,K3=GAIN (IGAIN=1 ONLY)
C*    C1,C2,C3=COEFF FOR K(1) --  ( IGAIN=2 ONLY )
C*    C4,C5,C6=COEFF FOR K(2) --  ( IGAIN=2 ONLY )
C*    C7,C8,C9=COEFF FOR K(3) --  ( IGAIN=2 ONLY )
C*    Z'S=RANGE MEASUREMENTS (IMEAS=1 ONLY)
C***************************************************************
      REAL IDENT(3,3),K(3),N(250)
      DIMENSION C(3),CON(3),PHI(3,3),PHIT(3,3),P(3,3),PP(3,3),PPP(3,3),
     1          R(250),RD(250),RDD(250),RMR(3),RMV(3),RMA(3),S(3,3),
     2          SUM1(3),SUM2(3),SUM3(3),TEM(3,3),TEMM(3),XH(3),XHP(3),
     3          KKKH(3,3),Z(250)
C*
      DATA ((IDENT(I,J),I=1,3),J=1,3)/1.,0.,0.,0.,1.,0.,0.,0.,1./
      DATA JJ/7777777777777B/
      IX=34158714927
      ONE=1.0
      TWO=2.0
      THREE=3.0
      DUM=0.
      R0=0.
      V0=0.
      A=0.
C*READ INPUT
 1    READ(5,5) IMEAS,IGAIN,INT1A,INT1B,INT2A,INT2B,TRATE,SECS
 5    FORMAT(6I5,2F10.1)
C*TEST END OF FILE
      IF(EOF(5).EQ.0)GO TO 20
      WRITE(6,15)
 15   FORMAT(1H1//10X,24H***NORMAL TERMINATION***)
      STOP
C*READ INPUT
 20   READ(5,25) SVSQ,SWSQ,R0,V0,A,(K(I),I=1,3)
 25   FORMAT(8F10.1)
      IF(IGAIN.EQ.2) READ(5,25)C1,C2,C3,C4,C5,C6,C7,C8,C9
```

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```
C*INITIALIZE
      XX=FLOAT(IX)
      CALL RANSET(XX)
      NPTS=SECS/TRATE+0.000001
      ONESEC=ONE/TRATE
      CON(1)=ONE/FLOAT(INT1B-INT1A+1)
      CON(2)=ONE/FLOAT(INT2B-INT2A+1)
      CON(3)=ONE/FLOAT(NPTS-3)
      SIG=SQRT(SVSQ)
      DO 30 I=1,3
      SUM1(I)=0.
      SUM2(I)=0.
   30 SUM3(I)=0.
      TEMP=SWSQ*TRATE/SVSQ*TRATE
      T1=TRATE
      T2=T1*T1
      T3=T1*T2
      T4=T1*T3
      S(1,1)=TEMP*T4
      S(2,1)=TEMP*THREE*T3
      S(3,1)=TEMP*6.0*T2
      S(1,2)=S(2,1)
      S(2,2)=TEMP*9.0*T2
      S(3,2)=TEMP*18.0*T1
      S(1,3)=S(3,1)
      S(2,3)=S(3,2)
      S(3,3)=TEMP*36.0
      IF(IMEAS.EQ.1) READ(5,25) (Z(KK),KK=1,NPTS)
      PHI(1,1)=ONE
      PHI(2,2)=ONE
      PHI(3,3)=ONE
      PHI(2,1)=0.
      PHI(3,1)=0.
      PHI(3,2)=0
      PHI(1,2)=T1
      PHI(2,3)=T1
      PHI(1,3)=T2/TWO
      DO 40 J=1,3
      DO 40 I=1,3
   40 PHIT(I,J)=PHI(J,I)
C* PRINT INPUT & HEADINGS
      IF(IMEAS-1)50,60,70
   50 WRITE(6,55)R0,V0,A
   55 FORMAT(1H1//5X,21HCONSTANT ACCELERATION,5X,8HR0,V0,A=,3E15.5)
      GO TO 80
   60 WRITE(6,65)
   65 FORMAT(1H1//5X,16HTRAJECTORY INPUT/)
      GO TO 80
   70 NN=2
      IF(IMEAS.EQ.3) NN=6
      WRITE(6,75) NN
   75 FORMAT(1H1//5X,12HMANUEVER IS ,I1,6HG TURN/)
   80 IF(IGAIN-1)90,100,110
   90 WRITE(6,95)
   95 FORMAT(/5X,16HGAIN IS COMPUTED)
      GO TO 120
  100 WRITE(6,105) (K(I),I=1,3)
```

```
  105   FORMAT(/5X,13HCONSTANT GAIN,5X,9HK1,K2,K3=,3E16.7)
        GO TO 120
  110   WRITE(6,115)C1,C4,C7,C2,C5,C8,C3,C6,C9
  115   FORMAT(/5X,27HSYNTHETIC TIME VARYING GAIN//10X,19HK(1)=(C1/(K+C2))
       1+C3,11X,21HK(2)=(C4/(K*K+C5))+C6,9X,23HK(3)=(C7/(K*K*K+C8))+C9,//1
       24X,3HC1=,E12.5,15X,3HC4=,E12.5,15X,3HC7=,E12.5/14X,3HC2=,E12.5,15X
       3,3HC5=,E12.5,15X,3HC8=,E12.5/14X,3HC3=,E12.5,15X,3HC6=,E12.5,15X,3
       4HC9=,E12.5/)
C*
  120   WRITE(6,125)FRATE,SECS,NPTS,SVSQ,SWSQ,INT1A,INT1B,INT2A,INT2B,NPTS
  125   FORMAT(/5X,11HSMPLG RATE=,F7.2,5X,8HTOT SEC=,F7.2,5X,7HNO PTS=,I3,
       15X,5HSVSQ=,F8.2,5X,5HSWSQ=,F8.2//5X,21HSTATISTICS INTERVALS:,5X,I3
       2,2H--,I3,5X,I3,2H--,I3,5X,3H4--,I3//2X,11HK      R(K),9X,5HRD(K),9
       3X,6HRDD(K),9X,5HXH(1),9X,5HXH(2),9X,5HXH(3),10X,2HK1,12X,2HK2,12X,
       +2HK3/)
C*
        DO 130 KK=1,NPTS
        N(KK)=0.
        RD(KK)=0.
  130   RDD(KK)=0.
        DO 140 I=1,3
        RMP(I)=0.
        RMV(I)=0.
  140   RMA(I)=0.
C*MAIN EXECUTION LOOP
        DO 500 KK=1,NPTS
        L=KK
        TT=FLOAT(L)
        T=TT/ONESEC
C*
        IF(IMEAS-1)150,160,170
  150   R(KK)=R0+V0*T+0.5*A*T*T
        RD(KK)=V0+A*T
        RDD(KK)=A
        GO TO 180
  160   R(KK)=Z(KK)
        GO TO 200
  170   ANG=0.031416*TT
        IF(IMEAS.EQ.3)GO TO 175
C* 2G TURN
        R(KK)=R0+24.355*(.04*TT-1.27324*SIN(ANG))
        RD(KK)=24.955*(ONE-COS(ANG))
        RDD(KK)=19.60*SIN(ANG)
        GO TO 180
C* 6G TURN
  175   R(KK)=R0+2.93466*(TT-31.831*SIN(ANG))
        RD(KK)=74.3665*(ONE-COS(ANG))
        RDD(KK)=58.8*SIN(ANG)
C*
  180   RNTOT=0.
        DO 190 J=1,12
        RN=RANF(DUM)
  190   RNTOT=RNTOT+RN
C*NOISE
        N(KK)=SIG*(RNTOT-6.0)
C*RANGE MEASUREMENT
        Z(KK)=R(KK)+N(KK)
```

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```
C*TEST PASS NUMBER
  200 IF(KK-3)200,210,220
  210 XH(1)=Z(3)
      XH(2)=(2.5*(THREE*Z(3)-4.0*Z(2)+Z(1))
      XH(3)=0.
      IF(IGAIN.GT.0)GO TO 410
      P(1,1)= VSQ
      P(2,1)= HREE*SVSQ/(TWO*T1)
      P(3,1)=SVSQ/T2
      P(1,2)=P(2,1)
      P(2,2)=13.0*SVSQ/(TWO*T2)
      P(3,2)=6.0*SVSQ/T3
      P(1,3)=P(3,1)
      P(2,3)=P(3,2)
      P(3,3)=6.0*SVSQ/T4+SWSQ
      GO TO 41
  220 DO 230 J=1,3
  230 XHP(J)=XH(J)
      IF(IGAIN-1)235,290,280
  235 DO 240 J=1,3
      DO 240 I=1,3
  240 PPP(I,J)=P(I,J)
C*FILTER EQUATION 1
      DO 250 J=1,3
      DO 250 I=1,3
  250 TEM(I,J)=PHI(I,1)*PPP(1,J)+PHI(I,2)*PPP(2,J)+PHI(I,3)*PPP(3,J)
      DO 260 J=1,3
      DO 260 I=1,3
  260 PP(I,J)=TEM(I,1)*PHIT(1,J)+TEM(I,2)*PHIT(2,J)+TEM(I,3)*PHIT(3,J)
      DO 270 J=1,3
      DO 270 I=1,3
  270 PP(I,J)=PP(I,J)+S(I,J)
C*FILTER EQUATION 2---GAIN
      IF(IGAIN.GT.0)GO TO 280
      TEMP=ONE/(PP(1,1)+SVSQ)
      K(1)=TEMP*PP(1,1)
      K(2)=TEMP*PP(1,2)
      K(3)=TEMP*PP(1,3)
      GO TO 290
  280 K(1)=(C1/(TT+C2))+C3
      K(2)=(C4/(TT*TT+C5))+C6
      K(3)=(C7/(TT*TT*TT+C8))+C9
C*FILTER EQUATION 3---ESTIMATES
  290 DO 300 I=1,3
  300 C(I)=PHI(I,1)*XHP(1)+PHI(I,2)*XHP(2)+PHI(I,3)*XHP(3)
      TEMPX=ONE*XHP(1)+T1*XHP(2)+(T2/TWO)*XHP(3)
      TEMPX=Z(KK)-TEMPX
      DO 310 I=1,3
  310 TEMM(I)=K(I)*TEMPX
      DO 320 I=1,3
  320 XH(I)=C(I)+TEMM(I)
      IF(IGAIN.GT.0)GO TO 420
C*FILTER EQUATION 4
      DO 330 I=1,3
  330 TEM(I,1)=PP(1, )
      DO 340 J=2,3
      DO 340 I=1,3
```

```
  340 TEM(I,J)=0.0
      DO 350 J=1,3
      DO 350 I=1,3
  350 XKKH(I,J)=TEMP*TEM(I,J)
      DO 360 J=1,3
      DO 360 I=1,3
  360 TEM(I,J)=IDENT(I,J)-XKKH(I,J)
      DO 370 J=1,3
      DO 370 I=1,3
  370 P(I,J)=TEM(I,1)*PP(1,J)+TEM(I,2)*PP(2,J)+TEM(I,3)*PP(3,J)
      GO TO 420
C*PRINT OUTPUT
  400 WRITE(6,450)KK,R(KK),RD(KK),RDD(KK)
      GO TO 500
  410 WRITE(6,450)KK,R(KK),RD(KK),RDD(KK),(XH(I),I=1,3)
      GO TO 500
  420 WRITE(6,450)KK,R(KK),RD(KK),RDD(KK),(XH(I),I=1,3),(K(I),I=1,3)
  450 FORMAT(1X,I3,9E14.6)
C*SUM FOR STATS
      IF(KK.LT.INT1A .OR. KK.GT.INT1B)GO TO 460
      SUM1(1)=SUM1(1)+(R(KK)-XH(1))**2
      IF(IMEAS.EQ.1)GO TO 460
      SUM2(1)=SUM2(1)+(RD(KK)-XH(2))**2
      SUM3(1)=SUM3(1)+(RDD(KK)-XH(3))**2
  460 IF(KK.LT.INT2A .OR. KK.GT.INT2B)GO TO 470
      SUM1(2)=SUM1(2)+(R(KK)-XH(1))**2
      IF(IMEAS.EQ.1)GO TO 470
      SUM2(2)=SUM2(2)+(RD (KK)-XH(2))**2
      SUM3(2)=SUM3(2)+(RDD(KK)-XH(3))**2
  470 IF(KK.LT.I16) GO TO 500
      SUM1(3)=SUM1(3)+(R(KK)-XH(1))**2
      IF(IMEAS.EQ.1)GO TO 500
      SUM2(3)=SUM2(3)+(RD(KK)-XH(2))**2
      SUM3(3)=SUM3(3)+(RDD(KK)-XH(3))**2
  500 CONTINUE
C*
      WRITE(6,540)
  540 FORMAT(1H1//5X,17HMEASUREMENT,NOISE/)
      DO 600 KK=1,NPTS
      II=KK
      WRITE(6,550)II,Z(KK),N(KK)
  550 FORMAT(5X,I5,5X,2E15.6)
  600 CONTINUE
C*
      DO 700 I=1,3
      RMR(I)=SQRT(CON(I)*SUM1(I))
      RMV(I)=SQRT(CON(I)*SUM2(I))
  700 RMA(I)=SQRT(CON(I)*SUM3(I))
C*
      WRITE(6,710)(RMR(I),RMV(I),RMA(I),I=1,3)
  710 FORMAT(1H1//2X,15HRMS DEVIATIONS://32X,5HRANGE,11X,3HVEL,11X,5HACC
     1EL/13X,9HINTERVAL1,3X,3E15.6/13X,9HINTERVAL2,3X,3E15.6/13X,9H PTS
     24--N,3X,3E15.6)
      GO TO 1
      END
```

DESCRIPTION

Computer program 'CNVEL' is a FORTRAN coded range type constant velocity Kalman Filter. The program contains the necessary equations to implement one of three models (trajectories): constant accelerations, straight line, sinusoidal acceleration (2g amplitude), and a sinusoidal acceleration (6g amplitude).

The program computes statistics which can be used to compare range, velocity and acceleration with their respective estimated values.

The purpose of the program is twofold -

1) to provide a means of evaluating several variables of the filter,
2) to enable the user to determine optimum values of some significant filter parameters.

METHOD

(1) Calculate 'true' values of range velocity and acceleration (R, $\dot{R}$, $\ddot{R}$) according to one of three methods:

a. Constant acceleration, straight line trajectory

$$R_i = X_1(k) = X_1(k-1) + TX_2(k-1) + \frac{T^2}{2} X_3(k-1)$$

$$\dot{R}_i = X_2(k) = X_2(k-1) + TX_3(k-1)$$

$$\ddot{R}_i = X_3(k) = \text{constant}$$

b. Sinusoidal acceleration, 2g amplitude

$$R_i = X_1(k) = 0.9982\ [k - 31.831 \sin (.031416k)] + R_o$$

$$\dot{R}_i = X_2(k) = 24.955\ [1.0 - \cos (.031416k)]$$

$$\ddot{R}_i = X_3(k) = 19.60 \sin (.031416k)$$

c. Sinusoidal acceleration, 6g amplitude

$$R_i = X_1(k) = 2.9947\ [k - 31.831 \sin (.031416k)] + R_o$$

$$\dot{R}_i = X_2(k) = 74.866\ [1.0 - \cos (.031416k)]$$

$$\ddot{R}_i = X_3(k) = 58.8 \sin (.031416k)$$

where T = sampling rate, sec., and

k = is an integer which takes on all values 1 through NPTS

$(NPTS = T_F - T_o)/\Delta t$

(2) Generation of Gaussian noise:

Generate 12 random numbers each having a uniform distribution between 0 and 1 - represented by RN_i

Then - noise = $n(k) = \alpha_v \left[\sum_1^{12} RN_i - 6.0 \right]$ is a quasi-gaussian random variable with zero mean and standard deviation = α_v

(3) The range measurement is $Z(k) = R(k) + n(k)$

(4) $\hat{X}_1$, $\hat{X}_2$, $\hat{X}_3$ = estimates of R, $\dot{R}$, $\ddot{R}$

To initialize:

$$\hat{X}_1 = Z(2)$$

$$\hat{X}_2 = 25.0\ [Z(2) - X(1)]$$

$$\hat{X}_3 = 0.0$$

(5) Filter Equations

a. Initialization

$$P(2/2) = 625\ \sigma_v^{\ 2} \begin{vmatrix} .0016 & .04 \\ .04 & 2.0 \end{vmatrix}$$

b. Iteration

$$P(k+1|k) = \phi P(k|k)\ \phi' + Q'$$

where:

$$\phi = \begin{vmatrix} 1 & .04 \\ 0 & 1 \end{vmatrix}$$

$$\phi' = \begin{vmatrix} 1 & 0 \\ .04 & 1 \end{vmatrix}$$

$$Q' = \sigma_w^{\ 2} \begin{vmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{vmatrix}$$

$$K(k+1) = \frac{1}{P_{11}(k+1|k) + \sigma_v^{\ 2}} \begin{bmatrix} P_{11}(k+1|k) \\ P_{21}(k+1|k) \end{bmatrix}$$

$$\hat{X}(k+1|k+1) = \phi\ \hat{X}(k|k) + K(k+1)\ Z(k+1) - (1.04)\hat{X}(k|k)$$

$$P(k+1|k+1) = \begin{bmatrix} 1-K_1(k+1) & 0 \\ -K_2(k+1) & 1 \end{bmatrix} P(k+1|k)$$

The following RMS deviations are computed:

$$(\Delta X_1)_{RMS} = \sqrt{\frac{1.0}{(N2-N1+1)} \sum_{k=N1}^{N2} \left[R(k) - \hat{X}_1(k|k)\right]^2}$$

$$(\Delta X_2)_{RMS} = \sqrt{\frac{1.0}{(N2-N1+1)} \sum_{k=N1}^{N2} \left[\dot{R}(k) - \hat{X}_2(k|k)\right]^2}$$

where $N1 \neq N2$ = initial and final indices of the points to be utilized in the computations of statistics

INPUT PREPARATION

CARD TYPE #1

VARIABLE	COLS.	TYPE	DEFINITION
R_o	1-10	F.Pt.	Initial range, meters
$\dot{R}_o$	11-20	F.Pt.	Initial velocity, meters/sec
$\ddot{R}_o$	21-30	F.Pt.	Initial acceleration, m/sec^2
T_o	31-40	F.Pt.	Initial time, sec
T_F	41-50	F.Pt.	Final time, sec
Δ_t	51-60	F.Pt.	Time increment, sec

CARD TYPE #2

$\sigma_w{}^2$	1-10	F.Pt.	Variance, maneuver noise
$\sigma_v{}^2$	11-20	F.Pt.	Variance, measurement noise
NEND	21-25	Int.	Value of index of highest point to be printed

CARD TYPE #3

NOST	1-5	Int.	Number of sets of statistics to be computed (3 in most cases)

CARD TYPE #4 (1 card per statistics set)

NST	1-5	Int.	Initial and final 'K' indices of points to be used in computation of statistics set
NFN	6-10	Int.	

OUTPUT

(1) Input values of R_o, R_o, R_o, σ_v^2, σ_w^2

(2) Iterative output at each 'k'

k, $R(k)$, $\dot{R}(k)$, $\ddot{R}(k)$, $\hat{x}_1(k)$, $\hat{x}_2(k)$, $\hat{x}_3(k)$,

xkA, xkB (Kalman gaon constants),

z(k)= measurement of range

xN(k)= noise

(3) Statistics: RMS deviations for position and velocity

```
      PROGRAM CNVEL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
     0DIMENSION R(300),RD(300),RDD(300),Z(300),XA(300),XB(300),
     1TT(300),XN(300)
      DATA JJ/77777777777776/
      TTT=0.1
      DUM=0.0
      TIM=TIME(TTT)
      IX=AND(TIM,JJ)
    1 READ(5,2)RO,ROD,RODD,TO,TF,DELT
    2 FORMAT(6F10.0)
      ICHK=EOF(5)
      IF (ICHK .NE. 0) GO TO 2000
  400 READ(5,401) ALF,BET,NEND
  401 FORMAT(2F10.0,I5)
      IF (ALF .LT. -.5) GO TO 1
      IX=34858714927
      XX=FLOAT(IX)
      CALL RANSET(XX)
      WRITE(6,11)
   11 FORMAT(1H1)
      WRITE(6,19) RO,ROD,RODD,ALF,BET
   19 0FORMAT(3H R=,E13.6,6H     V=,E13.6,6H     A=,E13.6,
     111H      SIGWSQ,E13.6,11H      SIGVSQ,E13.6)
      SN=(TF-TO)/DELT
      N=SN+1.0
      IF (N .GT. 300) N=300
      IF (N .LT. NEND) GO TO 400
      DO 3 I=1,N
      XI=I
      T=XI*DELT
      TT(I)=T
      RDD(I)=RODD
      RD(I)=RODD*(T-TO)+ROD
      R(I)=RO+ROD*(T-TO)+0.5*RODD*(T-TO)**2
      RNTOT=0.0
      DO 6 J=1,12
      RN=RANF(DUM)
      RNTOT=RNTOT+RN
    6 CONTINUE
      RNTOT=6.0*(RNTOT-6.0)
      XN(I)=RNTOT
      Z(I)=R(I)+XN(I)
    3 CONTINUE
      PA=BET
      PB=25.0*BET
      PC=25.0*BET
      PD=1250.0*BET
      XA(1)=Z(1)
      XA(2)=Z(2)
      XB(1)=0.0
      XB(2)=(Z(2)-Z(1))/DELT
      M=N-1
      WRITE(6,12)
   12 0FORMAT(6X,1HK,12X,1HR,12X,1HV,11X,4HX(1),9X,4HX(2),8X,
     15HXK(1),8X,5HXK(2),10X,1HZ,10X,5HNOISE)
      XKA=0.0
      XKB=0.0
      DO 4 K=1,M
      IF (K .GT. NEND) GO TO 25
      WRITE(6,13) K,R(K),RD(K),XA(K),XB(K),XKA,XKB,Z(K),XN(K)
   13 FORMAT(5X,I3,5X,8E13.6)
   25 CONTINUE
      IF (K .EQ. 1) GO TO 4
      QA=PA+0.04*PB+0.04*PC+0.0016*PD+0.00000064*ALF
      QB=PB+0.04*PD+0.000032*ALF
      QC=PC+0.04*PD+0.000032*ALF
      QD=PD+0.0016*ALF
      XKA=QA/(QA+BET)
      XKB=QB/(QA+BET)
      L=K+1
      XA(L)=XA(K)+0.04*XB(K)+XKA*Z(L)-XKA*(XA(K)+0.04*XB(K))
      XB(L)=XB(K)+XKB*Z(L)-XKB*(XA(K)+0.04*XB(K))
      PA=(1.0-XKA)*QA
      PB=(1.0-XKA)*QB
      PC=-XKB*QA+QC
      PD=-XKB*QB+QD
    4 CONTINUE
      READ(5,40) NOST
   40 FORMAT(I5)
      DO 41 K=1,NOST
```

```
      READ(5,42)NST,NFN
   42 FORMAT(2I5)
      RMR=0.0
      RMV=0.0
      DEN=NFN-NST+1
      DO 43 J=NST,NFN,1
      RMR=(R(J)-XA(J))**2/DEN+RMR
      RMV=(RD(J)-XB(J))**2/DEN+RMV
   43 CONTINUE
      RMR=SQRT(RMR)
      RMV=SQRT(RMV)
      WRITE(6,44) K,RMR,RMV,NST,NFN
   44 FORMAT(11H STAT. SET ,I5,8H POS RMS,E13.6,8H VEL RMS,E13.6,
     14H  K=,I5,3H TO,I5)
   41 CONTINUE
      GO TO 400
 2000 WRITE(6,2001)
 2001 FORMAT(18H NORMAL COMPLETION    )
      STOP
      END
```

APPENDIX C

PROGRAM FOR TARGET STATE MEASUREMENT

The attached documentation pertains to the program 'ERRORV' (velocity). The decks for two additional programs; 'ERRØR' (acceleration) and 'ERRØRR' (range) are also attached. The three programs are very similar. The following is a list of the distributions utilized in each of the three programs -

PROGRAM	VARIABLES	TYPE DISTRIBUTION	
'ERRØRV'	v_t, v_a	uniform	range: 200, 600 KTS
	v_t&v_a errors	normal	σ = 4.0 KTS, $\bar{x}$=0
	α_t, α_a	uniform	range: -5°, +20°
	α_t &α_a errors	normal	σ = 0.1°, $\bar{x}$=0
	roll angle(ϕ)	uniform	range: -90°, -60° & +60°, +90°
	pitch angle(θ)	uniform	range: -30°, +45° (75%) -90°, -30° (12.5%) +45°, +90° (12.5%)
	yaw angle(ψ)	unifrom	range: -180°, +180°
	sine, cosine errors	normal	α = 0.005, $\bar{x}$=0
'ERROR'	at_x, aa_x	uniform	σ = 32.16, $\bar{x}$=0
	at_y, aa_y	uniform	σ = 8.0, $\bar{x}$=0
	a-comp errors	normal	pdf=$-8.0+4e^{(x/2)}$where x=normal dist, σ=1.0&$\bar{x}$=0
	ϕ,θ,ψ	(same as above)	
	sine, cosine errors	(same as above)	
'ERRØRR'	ε,η	uniform	range: -15°, +15°
	ε,η errors	normal	σ = .0691°, $\bar{x}$=0
	range	uniform	range: 0, 2500 ft.
	range error	normal	σ = 4.0 ft., $\bar{x}$=0

COMPUTER PROGRAM 'ERRORV' CONTAINS THE MONTE CARLO CALCULATION OF THE ERROR DISTRIBUTION OF THE VELOCITY OF THE TARGET RELATIVE TO THE ATTACKER IN THE ATTACKERS BODY FRAME OF REFERENCE.

The Program Requires 56000 Words (Octal) of Storage on the CDC 6600.

INPUT

Variable	Card Cols.	Type	Definition
XX	1-10	Floating Point	Seed for Random Number Generator
NPASS	11-15	Interger	No. of Passes

OUTPUT

The Output Consists of Four Columns of Data.

Col 1	$(V_x' - V_x)$	Primed values include error terms, unprimed values = No Error Terms
2	$(V_y' - V_y)$	
3	$(V_z' - V_z)$	
4	$(V_{tot}' - V_{tot})$	

LINE NOS.	COMMENTS
7-11	Define Constants
12	READ: Seed for random number generator and number of passes
15	Test for end of file on Input Tape 5
16-18	If last case has been processed, print 'NORMAL TERMINATION' and stop·if another case is to be processed, proceed to statement No. 30
20-21	Print Input Values
23	Initialize Random Number Generator
25	Start Main Execution Loop
27-29	Select airspeeds v_t & v_a from a uniform distribution having the range 200-600 KTS
31-33	Select airspeed errors from a normal distribution having a σ=4.0 KTS and a MEAN=0
35-37	Select angles of attack α_t & α_a from a uniform distribution having a range -5° to +20° (convert to radians)
39-41	Select angle of attack errors from a normal distribution having a α=0.1° and MEAN=0. (Convert to radians)
43-48	Compute airspeed components, target and attacker

$$\begin{pmatrix} v_{tx} \\ v_{ty} \\ v_{tz} \end{pmatrix} = \begin{pmatrix} \cos\alpha_t \\ 0 \\ \sin\alpha_t \end{pmatrix} v_t \qquad \begin{pmatrix} v_{ax} \\ v_{ay} \\ v_{az} \end{pmatrix} \begin{pmatrix} \cos\alpha_a \\ 0 \\ \sin\alpha_a \end{pmatrix} v_a$$

50-52	Select yaw angles from a uniform distribution with the range -180° to 180° (convert to radians)
53-59	Select roll angles from a uniform distribution with the range =90° to -60° and +60° to +90° (convert to radians)

LINE NOS.	COMMENTS
60-71	Select pitch angles from uniform distribution with the range: -90° to +90° where -90° to -30° Occurs 12 1/2 % of the time -30° to +45° " 75% " " " +45° to +90° " 12 1/2% " " " (convert to radians)
73-80	Select sine & cosine errors from a normal distribution with σ= .005, MEAN=0.
82-84	Set-up 'Y' matrix terms
85	Compute elements of 'Y' matrix
91	Solve matrix equation, no error terms----
93-95	Set-up 'Y' matrix terms (errors included)
96	Compute elements of 'Y' matrix
97-99	Set-up 'X' matrix terms (error included)
100	Compute elements of 'X' matrix
101-110	Add error terms: $v_t' = v_t + \varepsilon$; $v_a' = v_a + \varepsilon$ $\alpha_t' = \alpha_t + \varepsilon$ $\alpha_a' = \alpha_a + \varepsilon$ $v_{tx}' = \cos\alpha_t' \; v_t'$, $v_{ax}' = \cos\alpha_a' \; v_a'$ $v_{ty}' = 0$, $v_{ay}' = 0$ $v_{tz}' = \sin\alpha_t' \; v_t'$, $v_{az}' = \sin\alpha_a' \; v_a'$
112	Solve matrix equation (with error terms)--- $v_v' = [X'] \quad [Y'] \quad [v_t'] - [v_a']$
114-115	Difference velocity components ($v_v' - v_v$)
116	Compute total velocity defference from sum of squares
117-119	Store ($v_{vx}' - V_{vx}$) , (Vvy' - Vvy) , (Vvz'- Vvz) and ($V_{v_{tot}}' - V_{v_{tot}}$) in output array
120	End Main Execution Loop
122-125	Print Output
126	Return to statement No. 1 for next case

SUBROUTINE 'RANDOM'

Purpose: To compute 'NRN' random numbers from either a uniform or normal distribution. Subroutine 'RANF' is utilized to generate a uniformly distributed random number (0 → 1)

IDIST=0		Uniform Distribution
C1 =		Additive Term (To Scale Range)
C2 =		Multiplier Term (To Scale Range)

IDIST=1		Normal Distribution
C1 =		$\underline{\sigma}$ = STD Deviation
C2 =		$\overline{X}$ = Mean

Method:

a) Uniform Distribution

Start Loop 'A'
i=1, NRN
↓
RN=RANF(DUM)
↓
RNS_i = (RN+C1) C2
↓
End Loop 'A'

b) Normal Distribution

Start Loop 'B'
i=1, NRN
↓
RNTØT=0
↓
Start Loop 'C'
j=1, 12
↓
RN=RANF(DUM)
↓
RNTOT=RNTØT+RN
↓
End Loop 'C'
↓
180 RNS_i=C1(RNTØT=6.0)+C2
↓
End Loop 'B'

SUBROUTINE 'ELEM'

Purpose: To compute matrix elements in one of two ways specified by 'N'

Method: N=0

$$
\begin{aligned}
x(1,1) &= \cos(\psi)\ \cos(\theta) \\
x(2,1) &= -\sin(\psi)\ \cos(\phi) + \cos(\psi)\ \sin(\theta)\ \sin(\phi) \\
x(3,1) &= \sin(\psi)\ \sin(\phi) + \cos(\psi)\ \sin(\theta)\ \cos(\phi) \\
x(1,2) &= \sin(\psi)\ \cos(\theta) \\
x(2,2) &= \cos(\psi)\ \cos(\phi) + \sin(\psi)\ \sin(\theta)\ \sin(\phi) \\
x(3,2) &= -\cos(\psi)\ \sin(\phi) + \sin(\psi)\ \sin(\theta)\ \cos(\phi) \\
x(1,3) &= -\sin(\theta) \\
x(2,3) &= \cos(\theta)\ \sin(\phi) \\
x(3,3) &= \cos(\theta)\ \cos(\phi)
\end{aligned}
$$

N=1

$$
\begin{aligned}
x(1,1) &= \cos(\psi)\ \cos(\theta) \\
x(2,1) &= \sin(\psi)\ \cos(\theta) \\
x(3,1) &= -\sin(\theta) \\
x(1,2) &= -\sin(\psi)\ \cos(\phi) + \cos(\psi)\ \sin(\theta)\ \sin(\phi) \\
x(2,2) &= \cos(\psi)\ \cos(\phi) + \sin(\psi)\ \sin(\theta)\ \sin(\phi) \\
z(3,2) &= \cos(\theta)\ \sin(\phi) \\
x(1,3) &= \sin(\psi)\ \sin(\phi) + \cos(\psi)\ \sin(\theta)\ \cos(\phi) \\
x(2,3) &= -\cos(\psi)\ \sin(\phi) + \sin(\psi)\ \sin(\theta)\ \cos(\phi) \\
x(3,3) &= \cos(\theta)\ \cos(\phi)
\end{aligned}
$$

SUBROUTINE 'SOLVE'

Purpose: Given the matrices X, Y, AT, AA--Solve for A:

A = X*Y*AT-AA   Where

X=3x3
Y=3x3
AT=3x1
AA=3x1

Method:

a) $C_i = Y_{i,1}\ AT_1 + Y_{i,2}\ AT_2 + Y_{i,3}\ AT_3 \quad (i=1,3)$

b) $D_i = X_{i,1}\ C_1 + X_{i,2}\ C_2 + X_{i,3}\ C3 \quad (i=1,3)$

c) $A_i = D_i - AA_i \quad (i=1,3)$

```
      PROGRAM ERRORV(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON RNS(10),DUM
      DIMENSION VELT(3),VELA(3),ANGT(3),ANGA(3),XANG(2),ESA(3),ECA(3),
     1          EST(3),ECT(3),T(6),R(6),VV(3),VVE(3),ADIF(3),AOUT(300,4)
     2          ,X(3,3),Y(3,3)
C*
      DUM=0.
      RAD=0.017453293
      G=32.16
      X1=0.14285714
      X2=0.89285714
    1 READ(5,10) XX,NPASS
   10 FORMAT(F10.1,I5)
C*TEST END OF FILE
      IF(EOF(5).EQ.0)GO TO 30
      WRITE(6,20)
   20 FORMAT(1H1//10X,24H***NORMAL TERMINATION***)
      STOP
C*PRINT INPUT
   30 WRITE(6,40)XX,NPASS
   40 FORMAT(1H1//5X,12HSEED FOR RN=,E13.6,5X,13HNO OF PASSES=,I5)
C*INITIALIZE
      CALL RANSET(XX)
C*MAIN EXECUTION LOOP
      DO 500 K=1,NPASS
C*SELECT AIRSPEEDS
      CALL RANDOM(0,2,0.5,400.)
      VT=RNS(1)
      VA=RNS(2)
C*SELECT AIRSPEED ERRORS
      CALL RANDOM(1,2,4.0,0.)
      VTE=RNS(1)
      VAE=RNS(2)
C*SELECT ANGLES OF ATTACK
      CALL RANDOM(0,2,-0.2,25.0)
      ALPT=RNS(1)*RAD
      ALPA=RNS(2)*RAD
C*SELECT ANG OF ATTK ERRORS
      CALL RANDOM(1,2,0.1,0.)
      ALPTE=RNS(1)*RAD
      ALPAE=RNS(2)*RAD
C*COMPUTE V-COMPS
      VELT(1)=COS(ALPT)*VT
      VELT(2)=0.0
      VELT(3)=SIN(ALPT)*VT
      VELA(1)=COS(ALPA)*VA
      VELA(2)=0.0
      VELA(3)=SIN(ALPA)*VA
C*SELECT ROLL,PITCH,YAW
      CALL RANDOM(0,2,-0.5,360.)
      ANGT(3)=RNS(1)*RAD
      ANGA(3)=RNS(2)*RAD
      CALL RANDOM(0,2,-0.5,1.)
      DO 65 L=1,2
      XS=1.0
      IF(RNS(L).LT.0.) XS=-1.0
   65 XANG(L)=60.*(RNS(L)+XS)
```

```
      ANGT(1)=XANG(1)*RAD
      ANGA(1)=XANG(2)*RAD
      CALL RANDOM(0,2,0.,1.)
      DO 69 L=1,2
      IF(RNS(L).LT.X1)GO TO 67
      IF(RNS(L).GE.X2)GO TO 68
      XANG(L)=100.*(RNS(L)-X1)-30.0
      GO TO 69
   67 XANG(L)=60.0*RNS(L)/X1-90.0
      GO TO 69
   68 XANG(L)=45.0*(RNS(L)-X2)/0.10714286+45.0
   69 CONTINUE
      ANGT(2)=XANG(1)*RAD
      ANGA(2)=XANG(2)*RAD
C*SELECT ERRORS--SIN,COS
      CALL RANDOM(1,6,0.005,0.)
      DO 70 I=1,3
      ESA(I)=RNS(I)
   70 ECA(I)=RNS(I+3)
      CALL RANDOM(1,6,0.005,0.)
      DO 80 I=1,3
      EST(I)=RNS(I)
   80 ECT(I)=RNS(I+3)
C*SET-UP MATRIX--NO ERROR
      DO 100 I=1,3
      T(I)=COS(ANGT(I))
  100 T(I+3)=SIN(ANGT(I))
      CALL ELEM(T,Y,1)
      DO 110 I=1,3
      R(I)=COS(ANGA(I))
  110 R(I+3)=SIN(ANGA(I))
      CALL ELEM(R,X,0)
C*SOLVE MATRIX EQUATION--NO ERROR
      CALL SOLVE(X,Y,VELT,VELA,VV)
C*SET-UP MATRIV--ERROR TERMS
      DO 120 I=1,3
      T(I)=T(I)+ECT(I)
  120 T(I+3)=T(I+3)+EST(I)
      CALL ELEM(T,Y,1)
      DO 130 I=1,3
      R(I)=R(I)+ECA(I)
  130 R(I+3)=R(I+3)+ESA(I)
      CALL ELEM(R,X,0)
      VT=VT+VTE
      VA=VA+VAE
      ALPT=ALPT+ALPTE
      ALPA=ALPA+ALPAE
      VELT(1)=COS(ALPT)*VT
      VELT(2)=0.0
      VELT(3)=SIN(ALPT)*VT
      VELA(1)=COS(ALPA)*VA
      VELA(2)=0.0
      VELA(3)=SIN(ALPA)*VA
C*SOLVE MATRIX EQUATION--ERROR TERMS
      CALL SOLVE(X,Y,VELT,VELA,VVE)
C*DIFFERENCE
      DO 150 I=1,3
```

```
      PROGRAM ERRORV      74/74    OPT=1                     FTN 4.5+414        10/20/77  18.06.39       PAGE      3

          150  ADIF(I)=VVE(I)-VV(I)
               ASUM=SQRT(ADIF(1)*ADIF(1)+ADIF(2)*ADIF(2)+ADIF(3)*ADIF(3))
               DO 170 I=1,3
          170  AOUT(K,I)=ADIF(I)
               AOUT(K,4)=ASUM
          500  CONTINUE
        C*
               WRITE(6,600)
          600  FORMAT(1H1//5X,26HX-COMP,Y-COMP,Z-COMP,TOTAL/)
               WRITE(6,610) ((AOUT(K,I),I=1,4),K=1,NPASS)
          610  FORMAT(5X,4E16.7)
               GO TO 1
               END
```

```
      SUBROUTINE ELEM(TA,X,N)
C*COMPUTE MATRIX ELEMENTS
C*INPUT***************
C*TA(1)=COS(PHI)
C*TA(2)=COS(THETA)
C*TA(3)=COS(PSI)
C*TA(4)=SIN(PHI)
C*TA(5)=SIN(THETA)
C*TA(6)=SIN(PSI)
C********************
      DIMENSION TA(6),X(3,3)
      IF(N.EQ.1)GO TO 10
C*
      X(1,1)=TA(3)*TA(2)
      X(2,1)=-TA(6)*TA(1)+TA(3)*TA(5)*TA(4)
      X(3,1)=TA(6)*TA(4)+TA(3)*TA(5)*TA(1)
      X(1,2)=TA(6)*TA(2)
      X(2,2)=TA(3)*TA(1)+TA(6)*TA(5)*TA(4)
      X(3,2)=-TA(3)*TA(4)+TA(6)*TA(5)*TA(1)
      X(1,3)=-TA(5)
      X(2,3)=TA(2)*TA(4)
      X(3,3)=TA(2)*TA(1)
      GO TO 100
C*
   10 X(1,1)=TA(3)*TA(2)
      X(2,1)=TA(6)*TA(2)
      X(3,1)=-TA(5)
      X(1,2)=-TA(6)*TA(1)+TA(3)*TA(5)*TA(4)
      X(2,2)=TA(3)*TA(1)+TA(6)*TA(5)*TA(4)
      X(3,2)=TA(2)*TA(4)
      X(1,3)=TA(6)*TA(4)+TA(3)*TA(5)*TA(1)
      X(2,3)=-TA(3)*TA(4)+TA(6)*TA(5)*TA(1)
      X(3,3)=TA(2)*TA(1)
  100 RETURN
      END
```

```
      SUBROUTINE RANDOM(IDIST,NRN,C1,C2)
C**INPUT**************************************
C*    NRN=NO. OF RN*S TO BE GENERATED
C*    IF UNIFORM DISTRIBUTION---
C*       IDIST=0
C*           C1=ADDITIVE TERM
C*           C2=MULTIPLIER TERM
C*    IF NORMAL DISTRIBUTION ---
C*       IDIST=1
C*           C1=STD DEV OF DISTRIBUTION
C*           C2=MEAN OF DISTRIBUTION
C*********************************************
      COMMON RNS(10),DUM
C*TEST DISTRIBUTION TYPE
      IF(IDIST.EQ.1)GO TO 50
C*UNIFORM DIST
      DO 20 I=1,NRN
      RN=RANF(DUM)
   20 RNS(I)=(RN+C1)*C2
      GO TO 100
C*NORMAL DIST
   50 DO 70 I=1,NRN
      RNTOT=0.
      DO 60 J=1,12
      RN=RANF(DUM)
   60 RNTOT=RNTOT+RN
      RNS(I)=C1*(RNTOT-6.0)+C2
   70 CONTINUE
  100 RETURN
      END
```

```
              SUBROUTINE SOLVE(X,Y,AT,AA,A)
        C*TO SOLVE THE MATRIX EQUATION:
        C*            A(3)=X(3,3)*(Y(3,3)*AT(3))-AA(3)
        C*
              DIMENSION X(3,3),Y(3,3),AT(3),AA(3),A(3),C(3),D(3)
        C*
              DO 10 I=1,3
           10 C(I)=Y(I,1)*AT(1)+Y(I,2)*AT(2)+Y(I,3)*AT(3)
              DO 20 I=1,3
           20 D(I)=X(I,1)*C(1)+X(I,2)*C(2)+X(I,3)*C(3)
              DO 30 I=1,3
           30 A(I)=D(I)-AA(I)
        C*
              RETURN
              END
```

APPENDIX D

RADAR LAG PROGRAM

The SDATA program requries two cards of input for each pass which is to be analyzed. The first data card contains the following input which directs the reading of the Sight Eval tape being used.

F10.1	- XMISN	- The mission number
I10	- IPAS	- The pass to be analyzed from the above mission
I10	- ITME	- That time during the pass at which computations are to begin (msec)
I10	- NPTS	- The number of data time intervals over which SDATA is to perform computations

Twenty-five Sight Eval data arrays are required as input to SDATA. Each of these arrays is smoothed by SDATA. The number of points to be used in each smooth must be specified. This is read in on the second data card.

25I2	- NFIT(I),I=1,25	- The number of points to be used in smoothing each of the 25 Sight Eval variable arrays input to SDATA.

```
      PROGRAM SDATA(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
      COMMON/PTS/XMISN,IPAS,ITME,NPTS
      COMMON/TEN/TR(3,3)
      COMMON/ALL/IT(200),AA(25,200),NFIT(25)
      DIMENSION IAR(24),IDER(24)
      DIMENSION A(50)
      DATA (IAR(I),I=1,24)/1,0,3,0,5,0,7,0,9,0,11,12,13,14,15,16,17,18,
     1            19,20,21,22,23,24/
      DATA (IDER(I),I=1,24)/1,0,1,0,1,0,1,0,1,15*0/
      C(1,6)=100.0
      C(100)=1000.0
      NGO=0
C
      CALL INPT(IN)
      IF(IN.EQ.1)GO TO 10
C*TERMINATE RUN
    3 REWIND1
      WRITE(6,4)
    4 FORMAT(1H1//10X,22H**NORMAL TERMINATION**)
      STOP
C
   10 NEOF=0
C*READ HEADER
   20 BUFFER IN(1,1)(A(1),A(5))
      IF(UNIT(1))30,40,50
C*GOOD READ
   30 NPAS=1
      XHDR=A(1)
      IFLAG=0
      IF(XMISN.EQ.XHDR)IFLAG=1
      GO TO 100
C*END OF FILE
   40 NEOF=NEOF+1
      IF(NEOF.NE.1)GO TO 43
      WRITE(6,41)
   41 FORMAT(//10X,21HEOF ENCOUNTERED---HDR)
      NGO=NGO+1
      IF (NGO .GT. 10) GO TO 3
      GO TO 10
   43 IF(NEOF.NE.2)GO TO 46
      WRITE(6,44)
   44 FORMAT(//10X,19HERROR---NEOF=2--HDR)
      GO TO 3
   46 WRITE(6,47)
   47 FORMAT(//10X,11HEND OF TAPE)
      GO TO 3
C*PARITY ERROR
   50 WRITE(6,51)
   51 FORMAT(//10X,18HPARITY ERROR---HDR)
      GO TO 3
C
C
  100 NEOF=0
C*READ DATA RECORD
  110 BUFFER IN(1,1)(A(1),A(50))
      IF(UNIT(1))120,150,180
C*GOOD READ
```

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```
  120 IF(IFLAG-1)100,130,135
  130 IF(NPAS.NE.IPAS)GO TO 100
      NO=0
      IFLAG=2
  135 ITIM=A(1)+.00001
      IF(ITIME.GT.ITIM)GO TO 100
      IF(NOR.GE.NPTS)GO TO 100
      NOR=NOR+1
C*STORE DATA---ONE POINT
      L=NOR
      TT(L)=A(1)/C1000
      AA(1,L)=A(7)
      AA(3,L)=A(25)
      AA(5,L)=A(26)
      AA(7,L)=A(32)/C1000
      AA(9,L)=A(33)/C1000
      AA(11,L)=A(14)
      AA(12,L)=A(15)
      AA(13,L)=A(16)
      AA(14,L)=A(22)
      AA(15,L)=A(24)
      AA(16,L)=A(23)
      AA(17,L)=A(6)
      AA(18,L)=A(20)
      AA(19,L)=A(17)
      AA(20,L)=A(18)
      AA(21,L)=A(19)
      AA(22,L)=A(3)
      AA(23,L)=A(5)
      AA(24,L)=A(8)
      GO TO 100
C*END OF FILE
  150 NEOF=NEOF+1
      IF(NEOF.EQ.1)GO TO 165
      IF(NEOF.EQ.2)GO TO 20
      WRITE(6,160)
  160 FORMAT(//10X,20HERROR---NEOF=3--DATA)
      GO TO 3
  165 NPAS=NPAS+1
      IF(IFLAG.NE.2)GO TO 110
C*COMPUTATIONS---END OF PASS
C*COMPUTATIONS---END OF PASS
      DO 1000 M=1,24
      IF(IAR(M).EQ.0)GO TO 1000
      NAR=IAR(M)
      CALL FILL(NAR,NPTS)
 1000 CONTINUE
      DO 1005 M=1,24
      NDER=IDER(M)
      NAR=M
      CALL SMOO(NDER,NAR,NPTS)
 1005 CONTINUE
      CALL XMATH(NPTS)
C
      CALL INPT(IN)
      IF(IN.NE.0)GO TO 3
C*TEST MISSION
```

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```
      IFLAG=0
      IF(XMISN.EQ.XHDR)IFLAG=1
      GO TO 11?
C*PARITY ERROR
  180 IF(IFLAG.EQ.2)GO TO 190
C
  185 WRITE(6,187)
  187 FORMAT(//1?X,20HPARITY ERROR IGNORED)
      GO TO 10?
C
  190 IF(NOR.GE.NPTS)GO TO 1??
      NOR=NOR+1
      GO TO 10?
C
      END
```

```
      SUBROUTINE INPT(IN)                                      ORIG   131
      COMMON/PTS/XMISN,IPAS,TIME,NPTS                          ORIG   132
      COMMON/ALL/TT(200),W(25,200),NFIT(25)                    ORIG   133
C                                                              ORIG   134
      IN=0                                                     ORIG   135
      READ(5,10) XMISN,IPAS,TIME,NPTS                          ORIG   136
   10 FORMAT(F10.1,4I10)                                       ORIG   137
C                                                              ORIG   138
      IF(EOF(5).EQ.0)GO TO 20                                  ORIG   139
C                                                              ORIG   140
      IN=1                                                     ORIG   141
      GO TO 50                                                 ORIG   142
C                                                              ORIG   143
   20 READ(5,25)(NFIT(I),I=1,25)                               ORIG   144
   25 FORMAT(25I2)                                             ORIG   145
C                                                              ORIG   146
      WRITE(6,40)XMISN,IPAS,TIME,NPTS                          ORIG   147
   40 FORMAT(  //5X,13HINPUT SUMMARY//11X,F8.1,6X,I2,5X,I8,5X,I3)  ORIG   148
C                                                              ORIG   149
C                                                              ORIG   150
   50 RETURN                                                   ORIG   151
      END                                                      ORIG   152
```

```
      SUBROUTINE FILL(NAR,NDP)                                ORIG   153
      COMMON/ALL/TT(200),AA(25,200),NFIT(25)                  ORIG   154
      J=0                                                     ORIG   155
      DO 1 I=1,NDP                                            ORIG   156
      TES=ABS(AA(NAR,I))                                      ORIG   157
      IF(TES.GT.0.000001)GO TO 4                              ORIG   158
      IF(J.EQ.1)GO TO 3                                       ORIG   159
      J=1                                                     ORIG   160
      JSAV=I                                                  ORIG   161
      IF(JSAV.NE.NDP)GO TO 1                                  ORIG   162
      JSA=JSAV+1                                              ORIG   163
      AA(NAR,JSAV)=AA(NAR,JSA)                                ORIG   164
      GO TO 100                                               ORIG   165
    3 IF(I.NE.NDP)GO TO 1                                     ORIG   166
      IF(JSAV.EQ.1)GO TO 100                                  ORIG   167
      JSA=JSAV-1                                              ORIG   168
      DO 5 K=JSAV,NDP                                         ORIG   169
      AA(NAR,K)=AA(NAR,JSA)                                   ORIG   170
    5 CONTINUE                                                ORIG   171
      GO TO 100                                               ORIG   172
    4 IF(J.EQ.0)GO TO 1                                       ORIG   173
      KSAV=I                                                  ORIG   174
      KSA=KSAV+1                                              ORIG   175
      IF(JSAV.EQ.1)GO TO 6                                    ORIG   176
      JSA=JSAV-1                                              ORIG   177
      AV=(AA(NAR,JSA)+AA(NAR,KSAV))/2.0                       ORIG   178
      DO 7 K=JSAV,KSA                                         ORIG   179
      AA(NAR,K)=AV                                            ORIG   180
    7 CONTINUE                                                ORIG   181
      J=0                                                     ORIG   182
      GO TO 1                                                 ORIG   183
    6 DO 8 K=1,KSA                                            ORIG   184
      AA(NAR,K)=AA(NAR,KSAV)                                  ORIG   185
    8 CONTINUE                                                ORIG   186
      J=0                                                     ORIG   187
    1 CONTINUE                                                ORIG   188
                                                              ORIG   189
  100 RETURN                                                  ORIG   190
      END                                                     ORIG   191
```

```
      SUBROUTINE SMOO(NDER,NAP,NOP)                       ORIG  192
      COMMON/ALL/TT(200),AA(25,200),NFIT(25)              ORIG  193
      DIMENSION S(4),AW(200)                              ORIG  194
      DO 7 I=1,NOP                                        ORIG  195
    7 AW(I)=AA(NAP,I)                                     ORIG  196
      NINC=NFIT(NAP)/2                                    ORIG  197
      NSTF=NINC+1                                         ORIG  198
      NFIN=NOP-NINC                                       ORIG  199
      IND=NAP+1                                           ORIG  200
      IF(NSTF.GT.NFIN)GO TO 1000                          ORIG  201
      DO 1 I=NSTF,NFIN                                    ORIG  202
      J=I-NINC                                            ORIG  203
      K=I+NINC                                            ORIG  204
      XN=NFIT(NAP)                                        ORIG  205
      DO 2 L=1,4                                          ORIG  206
    2 S(L)=0.                                             ORIG  207
      DO 3 L=J,K                                          ORIG  208
      S(1)=S(1)+TT(L)                                     ORIG  209
      S(2)=S(2)+TT(L)*AW(L)                               ORIG  210
      S(3)=S(3)+AW(L)                                     ORIG  211
      S(4)=S(4)+TT(L)*TT(L)                               ORIG  212
    3 CONTINUE                                            ORIG  213
      DEN=XN*S(4)-S(1)*S(1)                               ORIG  214
      XNUM=XN*S(2)-S(1)*S(3)                              ORIG  215
      CNUM=S(4)*S(3)-S(1)*S(2)                            ORIG  216
      XM=XNUM/DEN                                         ORIG  217
      CM=CNUM/DEN                                         ORIG  218
      AA(NAP,I)=XM*TT(I)+CM                               ORIG  219
      IF(NDER.EQ.0)GO TO 10                               ORIG  220
      AA(IND,I)=XM                                        ORIG  221
   10 CONTINUE                                            ORIG  222
      IF(J.NE.1)GO TO 4                                   ORIG  223
      DO 5 L=1,NINC                                       ORIG  224
      AA(NAP,L)=XM*TT(L)+CM                               ORIG  225
      IF(NDER.EQ.0)GO TO 5                                ORIG  226
      AA(IND,L)=XM                                        ORIG  227
    5 CONTINUE                                            ORIG  228
    4 IF(K.NE.NOP)GO TO 1                                 ORIG  229
      NO=NOP-NINC                                         ORIG  230
      DO 6 L=NO,NOP                                       ORIG  231
      AA(NAP,L)=XM*TT(L)+CM                               ORIG  232
      IF(NDER.EQ.0)GO TO 6                                ORIG  233
      AA(IND,L)=XM                                        ORIG  234
    6 CONTINUE                                            ORIG  235
    1 CONTINUE                                            ORIG  236
C                                                         ORIG  237
 1000 RETURN                                              ORIG  238
      END                                                 ORIG  239
```

```
      SUBROUTINE XMATH(NDP)                                          ORIG  240
      COMMON/ALL/TT(200),AA(25,200),NFIT(25)                         ORIG  241
      COMMON/TERM/TM(3,3)                                            ORIG  242
      DIMENSION SAX(200),SAY(200),SAZ(200),SA(200),SBX(200),SBY(200),ORIG  243
     1SBZ(200),SB(200),SCX(200),SCY(200),SCZ(200),SC(200),SDX(200),  ORIG  244
     2SDY(200),SDZ(200),SD(200),SEX(200),SEY(200),SEZ(200),SE(200)   ORIG  245
     3,TMM(100,3,3)                                                  ORIG  246
      PI=3.141592654                                                 ORIG  247
      RTD=180./PI                                                    ORIG  248
      DTR=1./RTD                                                     ORIG  249
      G=32.16                                                        ORIG  250
      T=TT(1)                                                        ORIG  251
      PR=AA(23,1)                                                    ORIG  252
      CALL ATMOS(PR,PMAC,ZA)                                         ORIG  253
      PMACH=AA(22,1)                                                 ORIG  254
      VFU=PMAC*PMACH                                                 ORIG  255
      X=0.                                                           ORIG  256
      Y=0.                                                           ORIG  257
      Z=-ZA                                                          ORIG  258
      ALFT=AA(17,1)                                                  ORIG  259
      BETT=AA(18,1)                                                  ORIG  260
      TEMP=COS(ALFT)                                                 ORIG  261
      U=VFU*TEMP*COS(BETT)                                           ORIG  262
      V=VFU*TEMP*SIN(BETT)                                           ORIG  263
      W=VFU*SIN(ALFT)                                                ORIG  264
      PSIT=AA(14,1)                                                  ORIG  265
      PCHT=AA(15,1)                                                  ORIG  266
      ROLT=AA(16,1)                                                  ORIG  267
      SPSI=SIN(PSIT/2.0)                                             ORIG  268
      SPCH=SIN(PCHT/2.0)                                             ORIG  269
      SROL=SIN(ROLT/2.0)                                             ORIG  270
      CPSI=COS(PSIT/2.0)                                             ORIG  271
      CPCH=COS(PCHT/2.0)                                             ORIG  272
      CROL=COS(ROLT/2.0)                                             ORIG  273
      EA=CPSI*CPCH*CROL+SPSI*SPCH*SROL                               ORIG  274
      EB=CPSI*CPCH*SROL-SPSI*SPCH*CROL                               ORIG  275
      EC=CPSI*SPCH*CROL+SPSI*CPCH*SROL                               ORIG  276
      ED=SPSI*CPCH*CROL-CPSI*SPCH*SROL                               ORIG  277
C                                                                    ORIG  278
      CALL TRANS(EA,EB,EC,ED,PSI,PCH,ROL)                            ORIG  279
C                                                                    ORIG  280
      CALL BTOI(U,V,W,XD,YD,ZD)                                      ORIG  281
C                                                                    ORIG  282
      UA=AA(19,1)                                                    ORIG  283
      VA=AA(20,1)                                                    ORIG  284
      WA=AA(21,1)                                                    ORIG  285
C                                                                    ORIG  286
      CALL BTOI(UA,VA,WA,XDD,YDD,ZDD)                                ORIG  287
      ZDD=ZDD+G                                                      ORIG  288
      P=AA(11,1)                                                     ORIG  289
      Q=AA(12,1)                                                     ORIG  290
      R=AA(13,1)                                                     ORIG  291
C                                                                    ORIG  292
      NDPM=NDP-1                                                     ORIG  293
      DO 1 I=2,NDP                                                   ORIG  294
      JJ=I                                                           ORIG  295
      JM=I-1                                                         ORIG  296
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```
      T=TT(I)                                                  ORIG   297
      J=I-1                                                    ORIG   298
      DELT=T-TT(J)                                             ORIG   299
      EAD=0.5*(-R*EB-Q*EC-S*ED)                                ORIG   300
      EBD=0.5*(P*EA+R*EC-Q*ED)                                 ORIG   301
      ECD=0.5*(Q*EA-R*EB+P*ED)                                 ORIG   302
      EDD=0.5*(R*EA+Q*EB-P*EC)                                 ORIG   303
C                                                              ORIG   304
      EA=EA+EAD*DELT                                           ORIG   305
      EB=EB+EBD*DELT                                           ORIG   306
      EC=EC+ECD*DELT                                           ORIG   307
      ED=ED+EDD*DELT                                           ORIG   308
C                                                              ORIG   309
      CALL TRANS(EA,EB,EC,ED,PSI,PCH,ROL)                      ORIG   310
      TEMP=DELT*DELT                                           ORIG   311
      X=X+XD*DELT+0.5*XDD*TEMP                                 ORIG   312
      Y=Y+YD*DELT+0.5*YDD*TEMP                                 ORIG   313
      Z=Z+ZD*DELT+0.5*ZDD*TEMP                                 ORIG   314
C                                                              ORIG   315
      XD=XD+XDD*DELT                                           ORIG   316
      YD=YD+YDD*DELT                                           ORIG   317
      ZD=ZD+ZDD*DELT                                           ORIG   318
C                                                              ORIG   319
      UA=AA(19,I)                                              ORIG   320
      VA=AA(20,I)                                              ORIG   321
      WA=AA(21,I)                                              ORIG   322
      VEL=SQRT(XD*XD+YD*YD+ZD*ZD)                              ORIG   323
C                                                              ORIG   324
      CALL BTOI(UA,VA,WA,XDD,YDD,ZDD)                          ORIG   325
      ZDD=ZDD+G                                                ORIG   326
      ACC=SQRT(XDD*XDD+YDD*YDD+ZDD*ZDD)                        ORIG   327
      RTA=AA(1,I)                                              ORIG   328
      RTAD=AA(2,I)                                             ORIG   329
      RA=AA(3,I)                                               ORIG   330
      RAD=AA(4,I)                                              ORIG   331
      RE=AA(5,I)                                               ORIG   332
      RED=AA(6,I)                                              ORIG   333
      FA=AA(7,I)                                               ORIG   334
      FAD=AA(8,I)                                              ORIG   335
      FE=AA(9,I)                                               ORIG   336
      FED=AA(10,I)                                             ORIG   337
C                                                              ORIG   338
      TEMP=COS(FE)                                             ORIG   339
      RTI=RTA*TEMP*COS(FA)                                     ORIG   340
      RTJ=RTA*TEMP*SIN(FA)                                     ORIG   341
      RTK=-RTA*SIN(FE)                                         ORIG   342
C                                                              ORIG   343
      CALL BTOI(RTI,RTJ,RTK,RTX,RTY,RTZ)                       ORIG   344
C                                                              ORIG   345
      TPX=X+RTX                                                ORIG   346
      TPY=Y+RTY                                                ORIG   347
      TPZ=Z+RTZ                                                ORIG   348
C                                                              ORIG   349
      TEM1=COS(FA)                                             ORIG   350
      TEM2=SIN(FA)                                             ORIG   351
      TEM3=SIN(FE)                                             ORIG   352
      TVIM=RTAD*TEMP*TEM1-RTA*TEMP*TEM2*FAD-RTA*TEM3*TEM1*FED  ORIG   353
```

```
      TVJM=RTAD*TEMP*TEM2+RTA*TEMP*TEM1*FAD-RTA*TEM3*TEM2*FED          ORIG    354
      TVKM=-RTAD*TEM3-RTA*TEMP*FED                                     ORIG    355
                                                                       ORIG    356
      P=AA(11,I)                                                       ORIG    357
      Q=AA(12,I)                                                       ORIG    358
      R=AA(13,I)                                                       ORIG    359
      TVI=TVIM+Q*RTK-R*RTJ                                             ORIG    360
      TVJ=TVJM+R*RTI-P*RTK                                             ORIG    361
      TVK=TVKM+P*RTJ-Q*RTI                                             ORIG    362
C                                                                      ORIG    363
      CALL BTOI(TVI,TVJ,TVK,RTADX,RTADY,RTADZ)                         ORIG    364
      TVX=XD+RTADX                                                     ORIG    365
      TVY=YD+RTADY                                                     ORIG    366
      TVZ=ZD+RTADZ                                                     ORIG    367
      TV=SQRT(TVX*TVX+TVY*TVY+TVZ*TVZ)                                 ORIG    368
      TEMPR=RTA*RTA                                                    ORIG    369
      WLSX=(RTY*(TVZ-ZD)-RTZ*(TVY-YD))/TEMPR                           ORIG    370
      WLSY=(RTZ*(TVX-XD)-RTX*(TVZ-ZD))/TEMPR                           ORIG    371
      WLSZ=(RTX*(TVY-YD)-RTY*(TVX-XD))/TEMPR                           ORIG    372
      WLS=SQRT(WLSX*WLSX+WLSY*WLSY+WLSZ*WLSZ)                          ORIG    373
                                                                       ORIG    374
      FI=TEMP*TEM1                                                     ORIG    375
      FJ=TEMP*TEM2                                                     ORIG    376
      FK=-TEM3                                                         ORIG    377
      RI=COS(RE)*COS(RA)                                               ORIG    378
      RJ=COS(RE)*SIN(RA)                                               ORIG    379
      RK=-SIN(RE)                                                      ORIG    380
      CALL BTOI(FI,FJ,FK,FIX,FJY,FKZ)                                  ORIG    381
      TERM=FI*RI+FJ*RJ+FK*RK                                           ORIG    382
      IF(TERM.GT.1.0)TERM=1.0                                          ORIG    383
      IF(TERM.LT.-1.0)TERM=-1.0                                        ORIG    384
      TH=ACOS(TERM)                                                    ORIG    385
      FID=-TEM3*TEM1*FED-TEMP*TEM2*FAD                                 ORIG    386
      FJD=-TEM3*TEM2*FED+TEMP*TEM1*FAD                                 ORIG    387
      FKD=-TEMP*FED                                                    ORIG    388
C                                                                      ORIG    389
      TEM1=SIN(RE)                                                     ORIG    390
      TEM2=COS(RA)                                                     ORIG    391
      TEM3=COS(RE)                                                     ORIG    392
      TEM4=SIN(RA)                                                     ORIG    393
      RID=-TEM1*TEM2*RED-TEM3*TEM4*RAD                                 ORIG    394
      RJD=-TEM1*TEM4*RED+TEM3*TEM2*RAD                                 ORIG    395
      RKD=-TEM3*RED                                                    ORIG    396
C                                                                      ORIG    397
      WFI=FJ*FKD-FK*FJD                                                ORIG    398
      WFJ=FK*FID-FI*FKD                                                ORIG    399
      WFK=FI*FJD-FJ*FID                                                ORIG    400
      WF=SQRT(WFI*WFI+WFJ*WFJ+WFK*WFK)                                 ORIG    401
      WRI=RJ*RKD-RK*RJD                                                ORIG    402
      WRJ=RK*RID-RI*RKD                                                ORIG    403
      WRK=RI*RJD-RJ*RID                                                ORIG    404
      WR=SQRT(WRI*WRI+WRJ*WRJ+WRK*WRK)                                 ORIG    405
      EIN=SIN(TH)                                                      ORIG    406
      IF(ABS(EIN).LT.1.0E-6)GO TO 2                                    ORIG    407
      RNI=(RJ*FK-RK*FJ)/EIN                                            ORIG    408
      RNJ=(RK*FI-RI*FK)/EIN                                            ORIG    409
      RNK=(RI*FJ-RJ*FI)/EIN                                            ORIG    410
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      C                                                             ORIG 411
    2   THR=(WFI-WFI)*RNI+(WFJ-WFJ)*RNJ+(WFK-WFK)*RNK               ORIG 412
        TES=WFI*RNI+WFJ*RNJ+WFK*RNK                                 ORIG 413
        IF (TES .LT. 0.0) TH=-TH                                    ORIG 414
      C                                                             ORIG 415
        CALL XITOB(UAG,VAG,WAG,XDO,YDO,ZDO)                         ORIG 416
        CALL XITOB(U,V,W,XD,YD,ZD)                                  ORIG 417
      C                                                             ORIG 418
        TEMP=VPU*VPU                                                ORIG 419
        WOSI=(V*WAG-W*VAG)/TEMP                                     ORIG 420
        WOSJ=(W*UAG-U*WAG)/TEMP                                     ORIG 421
        WOSK=(U*VAG-V*UAG)/TEMP                                     ORIG 422
        WOS=SQRT(WOSI*WOSI+WOSJ*WOSJ+WOSK*WOSK)                     ORIG 423
      C                                                             ORIG 424
        CALL BTOI(P,Q,R,WPX,WPY,WPZ)                                ORIG 425
      C                                                             ORIG 426
        WP=SQRT(WPX*WPX+WPY*WPY+WPZ*WPZ)                            ORIG 427
        THI=TH*RNI                                                  ORIG 428
        THJ=TH*RNJ                                                  ORIG 429
        THK=TH*RNK                                                  ORIG 430
      C                                                             ORIG 431
        CALL BTOI(THI,THJ,THK,THX,THY,THZ)                          ORIG 432
      C                                                             ORIG 433
        PPSI=PSI*RTD                                                ORIG 434
        PPCH=PCH*RTD                                                ORIG 435
        PROL=ROL*RTD                                                ORIG 436
        PSIT=AA(14,I)                                               ORIG 437
        PCHT=AA(15,I)                                               ORIG 438
        ROLT=AA(16,I)                                               ORIG 439
        PPSIT=PSIT*RTD                                              ORIG 440
        PPCHT=PCHT*RTD                                              ORIG 441
        PROLT=ROLT*RTD                                              ORIG 442
        DPSI=PPSI-PPSIT                                             ORIG 443
        DPCH=PPCH-PPCHT                                             ORIG 444
        DROL=PROL-PROLT                                             ORIG 445
      C                                                             ORIG 446
      C                                                             ORIG 447
        PR=AA(23,I)                                                 ORIG 448
        PMACH=AA(22,I)                                              ORIG 449
        CALL ATMOS(PR,PMAC,ZA)                                      ORIG 450
        VPUT=PMAC*PMACH                                             ORIG 451
        ZT=ZA                                                       ORIG 452
        DVPU=VPU-VPUT                                               ORIG 453
        DZ=Z-ZT                                                     ORIG 454
      C                                                             ORIG 455
      C                                                             ORIG 456
        ALF=ASIN(W/VPU)                                             ORIG 457
        BET=ASIN(V/(VPU*COS(ALF)))                                  ORIG 458
        PALF=ALF*RTD                                                ORIG 459
        PBET=BET*RTD                                                ORIG 460
        ALFT=AA(17,I)                                               ORIG 461
        BETT=AA(18,I)                                               ORIG 462
        PBETT=BETT*RTD                                              ORIG 463
        PALFT=ALFT*RTD                                              ORIG 464
        DALF=PALF-PALFT                                             ORIG 465
        DBET=PBET-PBETT                                             ORIG 466
      C                                                             ORIG 467
```

```
      PTIDT=EA(2?,I)                                        ORIG   468
C                                                           ORIG   469
      PWLSX=WLSX*RTD                                        ORIG   470
      PWLSY=WLSY*RTD                                        ORIG   471
      PWLSZ=WLSZ*RTD                                        ORIG   472
      PWLS=WLS*RTD                                          ORIG   473
C                                                           ORIG   474
C                                                           ORIG   475
      PWPX=WPX*RTD                                          ORIG   476
      PWPY=WPY*RTD                                          ORIG   477
      PWPZ=WPZ*RTD                                          ORIG   478
      PWP=WP*RTD                                            ORIG   479
      PP=P*RTD                                              ORIG   480
      PQ=Q*RTD                                              ORIG   481
      PR=R*RTD                                              ORIG   482
C                                                           ORIG   483
C                                                           ORIG   484
      CALL BTOI(WFI,WFJ,WFK,WFX,WFY,WFZ)                    ORIG   485
C                                                           ORIG   486
      PWFX=WFX*RTD                                          ORIG   487
      PWFY=WFY*RTD                                          ORIG   488
      PWFZ=WFZ*RTD                                          ORIG   489
      PWF=WF*RTD                                            ORIG   490
      PWFI=WFI*RTD                                          ORIG   491
      PWFJ=WFJ*RTD                                          ORIG   492
      PWFK=WFK*RTD                                          ORIG   493
C                                                           ORIG   494
C                                                           ORIG   495
      CALL BTOI(WOSI,WOSJ,WOSK,WOSX,WOSY,WOSZ)              ORIG   496
C                                                           ORIG   497
      PWOSI=WOSI*RTD                                        ORIG   498
      PWOSJ=WOSJ*RTD                                        ORIG   499
      PWOSK=WOSK*RTD                                        ORIG   500
      PWOS=WOS*RTD                                          ORIG   501
      PWOSX=WOSX*RTD                                        ORIG   502
      PWOSY=WOSY*RTD                                        ORIG   503
      PWOSZ=WOSZ*RTD                                        ORIG   504
C                                                           ORIG   505
C                                                           ORIG   506
      PTHX=THX*RTD                                          ORIG   507
      PTHY=THY*RTD                                          ORIG   508
      PTHZ=THZ*RTD                                          ORIG   509
      PTH=SQRT(PTHX**2+PTHY**2+PTHZ**2)                     ORIG   510
      PTHI=THI*RTD                                          ORIG   511
      PTHJ=THJ*RTD                                          ORIG   512
      PTHK=THK*RTD                                          ORIG   513
C                                                           ORIG   514
C                                                           ORIG   515
      TEMP=PWPX*PWPX+PWPY*PWPY+PWPZ*PWPZ                    ORIG   516
      COSTR=(PTHX*PWPX+PTHY*PWPY+PTHZ*PWPZ)/TEMP            ORIG   517
      TEM1=(PTHX-COSTR*PWPX)**2                             ORIG   518
      TEM2=(PTHY-COSTR*PWPY)**2                             ORIG   519
      TEM3=(PTHZ-COSTR*PWPZ)**2                             ORIG   520
      COSTRE=SQRT(TEM1+TEM2+TEM3)                           ORIG   521
      TEMP=PWLSX*PWLSX+PWLSY*PWLSY+PWLSZ*PWLSZ              ORIG   522
      CLOSI=(PTHX*PWLSX+PTHY*PWLSY+PTHZ*PWLSZ)/TEMP         ORIG   523
      TEM1=(PTHX-CLOSI*PWLSX)**2                            ORIG   524
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      TEM2=(PTHY-CLOSI*PWLSY)**2                                          ORIG      525
      TEM3=(PTHZ-CLOSI*PWLSZ)**2                                          ORIG      526
      CLOSIE=SQRT(TEM1+TEM2+TEM3)                                         ORIG      527
      TEMP=PWFX*PWFX+PWFY*PWFY+PWFZ*PWFZ                                  ORIG      528
      CRLTR=(PTHX*PWFX+PTHY*PWFY+PTHZ*PWFZ)/TEMP                          ORIG      529
      TEM1=(PTHX-CRLTR*PWFX)**2                                           ORIG      530
      TEM2=(PTHY-CRLTR*PWFY)**2                                           ORIG      531
      TEM3=(PTHZ-CRLTR*PWFZ)**2                                           ORIG      532
      CRLTRE=SQRT(TEM1+TEM2+TEM3)                                         ORIG      533
      TEMP=PWOSX*PWOSX+PWOSY*PWOSY+PWOSZ*PWOSZ                            ORIG      534
      CTRTR=(PTHX*PWOSX+PTHY*PWOSY+PTHZ*PWOSZ)/TEMP                       ORIG      535
      TEM1=(PTHX-CTRTR*PWOSX)**2                                          ORIG      536
      TEM2=(PTHY-CTRTR*PWOSY)**2                                          ORIG      537
      TEM3=(PTHZ-CTRTR*PWOSZ)**2                                          ORIG      538
      CTRTRE=SQRT(TEM1+TEM2+TEM3)                                         ORIG      539
                                                                          ORIG      540
                                                                          ORIG      541
      TEMP=(RTX*XD+RTY*YD+RTZ*ZD)/(RTA*VPU)                               ORIG      542
      IF(TEMP.GT.1.0)TEMP=1.0                                             ORIG      543
      IF(TEMP.LT.-1.0)TEMP=-1.0                                           ORIG      544
      ZETA=ACOS(TEMP)*RTD                                                 ORIG      545
      TSK=TVX*XD+TVY*YD+TVZ*ZD                                            ORIG      546
      IF(TSK.LT.0.0)ZETA=-ZETA                                            ORIG      547
      TEMP=(RTX*TVX+RTY*TVY+RTZ*TVZ)/(TV*RTA)                             ORIG      548
      IF(TEMP.GT.1.0)TEMP=1.0                                             ORIG      549
      IF(TEMP.LT.-1.0)TEMP=-1.0                                           ORIG      550
      YETA=ACOS(TEMP)*RTD                                                 ORIG      551
      TAZ=ATAN(TVY/TVX)                                                   ORIG      552
      TEL=ASIN(TVZ/TV)                                                    ORIG      553
      PAZ=ATAN(YD/XD)                                                     ORIG      554
      PEL=ASIN(ZD/VPU)                                                    ORIG      555
      RAZ=ATAN(RTY/RTX)                                                   ORIG      556
      REL=ASIN(RTZ/RTA)                                                   ORIG      557
      IF (TAZ .LT. 0.0) TAZ=2.0*PI+TAZ                                    ORIG      558
      IF (PAZ .LT.0.0) PAZ=2.0*PI+PAZ                                     ORIG      559
      IF (RAZ .LT. 0.0) RAZ=2.0*PI+RAZ                                    ORIG      560
      PTAZ=TAZ*RTD                                                        ORIG      561
      PTEL=TEL*RTD                                                        ORIG      562
      PPAZ=PAZ*RTD                                                        ORIG      563
      PPEL=PEL*RTD                                                        ORIG      564
      PRAZ=RAZ*RTD                                                        ORIG      565
      PREL=REL*RTD                                                        ORIG      566
      SAX(JM)=FI                                                          NOWA        1
      SAY(JM)=FJ                                                          NOWA        2
      SAZ(JM)=FK                                                          NOWA        3
      SA(JM)=1.0                                                          ORIG      570
      SBX(JM)=FI                                                          ORIG      571
      SBY(JM)=FJ                                                          ORIG      572
      SBZ(JM)=FK                                                          ORIG      573
      SB(JM)=1.0                                                          ORIG      574
      SCX(JM)=WFX                                                         ORIG      575
      SCY(JM)=WFY                                                         ORIG      576
      SCZ(JM)=WFZ                                                         ORIG      577
      SC(JM)=WF                                                           ORIG      578
      SDX(JM)=WOSX                                                        ORIG      579
      SDY(JM)=WOSY                                                        ORIG      580
      SDZ(JM)=WOSZ                                                        ORIG      581
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      SD(JM)=WDS                                                           ORIG   582
      SEX(JM)=THX                                                          ORIG   583
      SEY(JM)=THY                                                          ORIG   584
      SEZ(JM)=THZ                                                          ORIG   585
      SE(JM)=PTH*DTR                                                       ORIG   586
      DO 48 LI=1,3                                                         ORIG   587
      DO 48 LJ=1,3                                                         ORIG   588
      TRM(JM,LI,LJ)=TR(LI,LJ)                                              ORIG   589
   48 CONTINUE                                                             ORIG   590
    1 CONTINUE                                                             ORIG   591
      SUAA=0.0                                                             ORIG   592
      SUAB=0.0                                                             ORIG   593
      SUAC=0.0                                                             ORIG   594
      SUAD=0.0                                                             ORIG   595
      SUAE=0.0                                                             ORIG   596
      SUBB=0.0                                                             ORIG   597
      SUBC=0.0                                                             ORIG   598
      SUBD=0.0                                                             ORIG   599
      SUBE=0.0                                                             ORIG   600
      SUCC=0.0                                                             ORIG   601
      SUCD=0.0                                                             ORIG   602
      SUCE=0.0                                                             ORIG   603
      SUDD=0.0                                                             ORIG   604
      SUDE=0.0                                                             ORIG   605
      SU3A=0.0                                                             ORIG   606
      SU4A=0.0                                                             ORIG   607
      SUAA=0.0                                                             ORIG   608
      SASM=0.0                                                             ORIG   609
      HTX=0.0                                                              ORIG   610
      HTY=0.0                                                              ORIG   611
      HTZ=0.0                                                              ORIG   612
      HWAX=0.0                                                             ORIG   613
      HWAY=0.0                                                             ORIG   614
      HWAZ=0.0                                                             ORIG   615
      ITES=3                                                               NOWA     4
      DO 30 M=1,5                                                          ORIG   617
      ITES=ITES+1                                                          ORIG   618
      IF (ITES .GE. NDPM) GO TO 30                                         ORIG   619
      MND=NDPM-ITES                                                        ORIG   620
      MDN=NDPM-ITES                                                        ORIG   621
      SUMF=0.0                                                             ORIG   622
      SUMM=0.0                                                             ORIG   623
      SUMN=0.0                                                             ORIG   624
      DO 31 KK=1,MND,1                                                     ORIG   625
      SUMF=SUMF+SE(KK)                                                     ORIG   626
      KL=KK+ITES                                                           ORIG   627
      FIX=TRM(KL,1,1)*SAX(KK)+TRM(KL,1,2)*SAY(KK)+TRM(KL,1,3)*SAZ(KK)      NOWA     5
      FJY=TRM(KL,2,1)*SAX(KK)+TRM(KL,2,2)*SAY(KK)+TRM(KL,2,3)*SAZ(KK)      NOWA     6
      FKZ=TRM(KL,3,1)*SAX(KK)+TRM(KL,3,2)*SAY(KK)+TRM(KL,3,3)*SAZ(KK)      NOWA     7
      FIX=TRM(KL,1,1)*SBX(KL)+TRM(KL,1,2)*SBY(KL)+TRM(KL,1,3)*SBZ(KL)      ORIG   635
      FJY=TRM(KL,2,1)*SBX(KL)+TRM(KL,2,2)*SBY(KL)+TRM(KL,2,3)*SBZ(KL)      ORIG   636
      FKZ=TRM(KL,3,1)*SBX(KL)+TRM(KL,3,2)*SBY(KL)+TRM(KL,3,3)*SBZ(KL)      ORIG   637
      ASG=FIX*FIX+FJY*FJY+FKZ*FKZ                                          NOWA     8
      IF (ASG .LT. -1.0) ASG=-1.0                                          ORIG   639
      IF (ASG .GT. 1.0) ASG=1.0                                            ORIG   640
      SUMN=SUMN+ACOS(ASG)                                                  ORIG   641
   31 CONTINUE                                                             ORIG   642
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      XDN=MDN                                                          ORIG 643
      SUMF=(SUMF/XDN)*1000.0                                           ORIG 644
      SUMM=(SUMM/XDN)*1000.0                                           ORIG 645
      SUMN=(SUMN/XDN)*1000.0                                           ORIG 646
      AG=ITES                                                          ORIG 647
      AG=AG*0.04                                                       ORIG 648
      WRITE(6,32)SUMF,SUMN,SUMM,AG,ITES                                ORIG 649
   32 FORMAT(20H LAG,LAG1,LAG2,DELAY,4E13.6,I5)                        ORIG 650
   30 CONTINUE                                                         ORIG 651
      DO 16 K=1,NDPM                                                   ORIG 652
      SUAA=SAX(K)**2+SAY(K)**2+SAZ(K)**2+SUAA                          ORIG 653
      SUBB=SBX(K)**2+SBY(K)**2+SBZ(K)**2+SUBB                          ORIG 654
      SUCC=SCX(K)**2+SCY(K)**2+SCZ(K)**2+SUCC                          ORIG 655
      SUDD=SDX(K)**2+SDY(K)**2+SDZ(K)**2+SUDD                          ORIG 656
      SUAB=SAX(K)*SBX(K)+SAY(K)*SBY(K)+SAZ(K)*SBZ(K)+SUAB              ORIG 657
      SUAC=SAX(K)*SCX(K)+SAY(K)*SCY(K)+SAZ(K)*SCZ(K)+SUAC              ORIG 658
      SUAD=SAX(K)*SDX(K)+SAY(K)*SDY(K)+SAZ(K)*SDZ(K)+SUAD              ORIG 659
      SUBC=SBX(K)*SCX(K)+SBY(K)*SCY(K)+SBZ(K)*SCZ(K)+SUBC              ORIG 660
      SUBD=SBX(K)*SDX(K)+SBY(K)*SDY(K)+SBZ(K)*SDZ(K)+SUBD              ORIG 661
      SUCD=SCX(K)*SDX(K)+SCY(K)*SDY(K)+SCZ(K)*SDZ(K)+SUCD              ORIG 662
      SUAE=SAX(K)*SEX(K)+SAY(K)*SEY(K)+SAZ(K)*SEZ(K)+SUAE              ORIG 663
      SUBE=SBX(K)*SEX(K)+SBY(K)*SEY(K)+SBZ(K)*SEZ(K)+SUBE              ORIG 664
      SUCE=SCX(K)*SEX(K)+SCY(K)*SEY(K)+SCZ(K)*SEZ(K)+SUCE              ORIG 665
      SUDE=SDX(K)*SEX(K)+SDY(K)*SEY(K)+SDZ(K)*SEZ(K)+SUDE              ORIG 666
      SU3A=SAX(K)**3+SAY(K)**3+SAZ(K)**3+SU3A                          ORIG 667
      SU4A=SAX(K)**4+SAY(K)**4+SAZ(K)**4+SU4A                          ORIG 668
      SUAEE=SA(K)*(SAX(K)*SEX(K)+SAY(K)*SEY(K)+SAZ(K)*SEZ(K))+SUAEE    ORIG 669
      HTX=HTX+SEX(K)                                                   ORIG 670
      HTY=HTY+SEY(K)                                                   ORIG 671
      HTZ=HTZ+SEZ(K)                                                   ORIG 672
      HWAX=HWAX+SAX(K)                                                 ORIG 673
      HWAY=HWAY+SAY(K)                                                 ORIG 674
      HWAZ=HWAZ+SAZ(K)                                                 ORIG 675
   16 CONTINUE                                                         ORIG 676
      DDELL=SUAA*(SUBB*(SUCC*SUDD-SUCD**2)-SUBC*(SUBC*SUDD-SUCD*SUBD)+ ORIG 677
     1SUBD*(SUBC*SUCD-SUCC*SUBD))-SUAB*(SUAB*(SUCC*SUDD-SUCD*SUCD)-    ORIG 678
     2SUBC*(SUAC*SUDD-SUCD*SUAD)+SUBD*(SUAC*SUCD-SUCC*SUAD))+SUAC*     ORIG 679
     3(SUAB*(SUBC*SUDD-SUCD*SUBD)-SUBB*(SUAC*SUDD-SUCD*SUAD)+SUBD*     ORIG 680
     4(SUAC*SUBD-SUBC*SUAD))-SUAD*(SUAB*(SUBC*SUCD-SUCC*SUBD)-SUBB*    ORIG 681
     5(SUAC*SUCD-SUCC*SUAD)+SUBC*(SUAC*SUBD-SUBC*SUAD))                ORIG 682
      DDELA=SUAE*(SUBB*(SUCC*SUDD-SUCD*SUCD)-SUBC*(SUBC*SUDD-SUCD*SUBD) ORIG 683
     1+SUBD*(SUBC*SUCD-SUCC*SUBD))-SUAB*(SUBE*(SUCC*SUDD-SUCD*SUCD)-   ORIG 684
     2SUBC*(SUCE*SUDD-SUCD*SUDE)+SUBD*(SUCE*SUCD-SUCC*SUDE))+SUAC*     ORIG 685
     3(SUBE*(SUBC*SUDD-SUBD*SUCD)-SUBB*(SUCE*SUDD-SUCD*SUDE)+SUBD*     ORIG 686
     4(SUCE*SUBD-SUBC*SUDE))-SUAD*(SUBE*(SUBC*SUCD-SUCC*SUBD)-SUBB*    ORIG 687
     5(SUCE*SUCD-SUCC*SUDE)+SUBC*(SUCE*SUBD-SUBC*SUDE))                ORIG 688
      DDELB=SUAA*(SUBE*(SUCC*SUDD-SUCD*SUCD)-SUBC*(SUCE*SUDD-SUCD*SUDE) ORIG 689
     1+SUBD*(SUCE*SUCD-SUCC*SUDE))-SUAE*(SUAB*(SUCC*SUDD-SUCD*SUCD)-   ORIG 690
     2SUBC*(SUAC*SUDD-SUCD*SUAD)+SUBD*(SUAC*SUCD-SUCC*SUAD))+SUAC*     ORIG 691
     3(SUAB*(SUCE*SUDD-SUCD*SUDE)-SUBE*(SUAC*SUDD-SUCD*SUAD)+SUBD*     ORIG 692
     4(SUAC*SUDE-SUCE*SUAD))-SUAD*(SUAB*(SUCE*SUCD-SUCC*SUDE)-SUBE*    ORIG 693
     5(SUAC*SUCD-SUCC*SUAD)+SUBC*(SUAC*SUDE-SUCE*SUAD))                ORIG 694
      DDELC=SUAA*(SUBB*(SUCE*SUDD-SUCD*SUDE)-SUBE*(SUBC*SUDD-SUCD*SUBD) ORIG 695
     1+SUBD*(SUBC*SUDE-SUCE*SUBD))-SUAB*(SUAB*(SUCE*SUDD-SUCD*SUDE)-   ORIG 696
     2SUBE*(SUAC*SUDD-SUCD*SUAD)+SUBD*(SUAC*SUDE-SUCE*SUAD))+SUAE*     ORIG 697
     3(SUAB*(SUBC*SUDD-SUCD*SUBD)-SUBB*(SUAC*SUDD-SUCD*SUAD)+SUBD*     ORIG 698
     4(SUAC*SUBD-SUBC*SUAD))-SUAD*(SUAB*(SUBC*SUDE-SUCE*SUBD)-SUBB*    ORIG 699
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      5(SUAC*SUDE-SUCE*SUAD)+SUBE*(SUAC*SUBD-SUBC*SUAD))                    ORIG   700
      DELD=SUAA*(SUBA*(SUCC*SUDE-SUCE*SUCD)-SUBC*(SUBC*SUDE-SUCE*SUBD)      ORIG   701
     1+SUBE*(SUBC*SUCD-SUCC*SUBD))-SUAB*(SUAB*(SUCC*SUDE-SUCE*SUCD)-        ORIG   702
     2SUBC*(SUAC*SUDE-SUCE*SUAD)+SUBE*(SUAC*SUCD-SUCC*SUAD))+SUAC*          ORIG   703
     3(SUAB*(SUBC*SUDE-SUCE*SUBD)-SUBB*(SUAC*SUDE-SUCE*SUAD)+SUBE*          ORIG   704
     4(SUAC*SUBD-SUBC*SUAD))-SUAE*(SUAB*(SUBC*SUCD-SUCC*SUBD)-SUBB*         ORIG   705
     5(SUAC*SUCD-SUCC*SUAD)+SUBC*(SUAC*SUBD-SUBC*SUAD))                    ORIG   706
      AKOF=DELA/DELL                                                       ORIG   707
      BKOF=DELB/DELL                                                       ORIG   708
      CKOF=DELC/DELL                                                       ORIG   709
      DKOF=DELD/DELL                                                       ORIG   710
      DILL=SUBB*SUCC-SUBC*SUBC                                             ORIG   711
      DILB=SUBE*SUCC-SUCE*SUBC                                             ORIG   712
      DILC=SUBB*SUCE-SUBE*SUBC                                             ORIG   713
      BKF=DILB/DILL                                                        ORIG   714
      CKF=DILC/DILL                                                        ORIG   715
      ALN=SUAE/SUAA                                                        ORIG   716
      BLN=SUBE/SUBB                                                        ORIG   717
      CLN=SUCE/SUCC                                                        ORIG   718
      DLN=SUDE/SUDD                                                        ORIG   719
      ALMEN=0.0                                                            ORIG   720
      ALSTD=0.0                                                            ORIG   721
      DIMEN=0.0                                                            ORIG   722
      DISTD=0.0                                                            ORIG   723
      AMEN=0.0                                                             ORIG   724
      ASTD=0.0                                                             ORIG   725
      BMEN=0.0                                                             ORIG   726
      BSTD=0.0                                                             ORIG   727
      CMEN=0.0                                                             ORIG   728
      CSTD=0.0                                                             ORIG   729
      DMEN=0.0                                                             ORIG   730
      DSTD=0.0                                                             ORIG   731
      XNDPM=NDPM                                                           ORIG   732
      THMEN=0.0                                                            ORIG   733
      THSTD=0.0                                                            ORIG   734
      DON=SUAA*SUAA-SU3A**2                                                ORIG   735
      AON=SUAAE*SUAA-SUAE*SU3A                                             ORIG   736
      BON=SUAE*SUAA-SUAAE*SU3A                                             ORIG   737
      AOA=AON/DON                                                          ORIG   738
      AOB=BON/DON                                                          ORIG   739
      HAN=XNDPM*SUAE-(HTX*HWAX+HTY*HWAY+HTZ*HWAZ)                          ORIG   740
      HAD=XNDPM*SUAA-(HWAX**2+HWAY**2+HWAZ**2)                             ORIG   741
      HA=HAN/HAD                                                           ORIG   742
      HBX=(HTX-HA*HWAX)/XNDPM                                              ORIG   743
      HBY=(HTY-HA*HWAY)/XNDPM                                              ORIG   744
      HBZ=(HTZ-HA*HWAZ)/XNDPM                                              ORIG   745
      HB=SQRT(HBX**2+HBY**2+HBZ**2)                                        ORIG   746
      HAMN=0.0                                                             ORIG   747
      HASD=0.0                                                             ORIG   748
      AQMN=0.0                                                             ORIG   749
      AQSD=0.0                                                             ORIG   750
      DO 17 K=1,NDPM                                                       ORIG   751
      ALFX=AKOF*SAX(K)+BKOF*SBX(K)+CKOF*SCX(K)+DKOF*SDX(K)                 ORIG   752
      ALFY=AKOF*SAY(K)+BKOF*SBY(K)+CKOF*SCY(K)+DKOF*SDY(K)                 ORIG   753
      ALFZ=AKOF*SAZ(K)+BKOF*SBZ(K)+CKOF*SCZ(K)+DKOF*SDZ(K)                 ORIG   754
      ALINC=(SEX(K)-ALFX)**2+(SEY(K)-ALFY)**2+(SEZ(K)-ALFZ)**2             ORIG   755
      ALMEN=ALMEN+SQRT(ALINC)                                              ORIG   756
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      ALSTD=ALSTD+ALINC                                          ORIG    757
      ALFX=BKF*SBX(K)+CKF*SCX(K)                                 ORIG    758
      ALFY=BKF*SBY(K)+CKF*SCY(K)                                 ORIG    759
      ALFZ=BKF*SBZ(K)+CKF*SCZ(K)                                 ORIG    760
      ALINC=(SEX(K)-ALFX)**2+(SEY(K)-ALFY)**2+(SEZ(K)-ALFZ)**2   ORIG    761
      DIMEN=DIMEN+SQRT(ALINC)                                    ORIG    762
      DISTD=DISTD+ALINC                                          ORIG    763
      ALINC=(SEX(K)-ALN*SAX(K))**2+(SEY(K)-ALN*SAY(K))**2+       ORIG    764
     1(SEZ(K)-ALN*SAZ(K))**2                                     ORIG    765
      AMEN=AMEN+SQRT(ALINC)                                      ORIG    766
      ASTD=ASTD+ALINC                                            ORIG    767
      ALINC=(SEX(K)-BLN*SBX(K))**2+(SEY(K)-BLN*SBY(K))**2+       ORIG    768
     1(SEZ(K)-BLN*SBZ(K))**2                                     ORIG    769
      BMEN=BMEN+SQRT(ALINC)                                      ORIG    770
      BSTD=BSTD+ALINC                                            ORIG    771
      ALINC=(SEX(K)-CLN*SCX(K))**2+(SEY(K)-CLN*SCY(K))**2+       ORIG    772
     1(SEZ(K)-CLN*SCZ(K))**2                                     ORIG    773
      CMEN=CMEN+SQRT(ALINC)                                      ORIG    774
      CSTD=CSTD+ALINC                                            ORIG    775
      ALINC=SE(K)**2                                             ORIG    776
      THMEN=THMEN+SQRT(ALINC)                                    ORIG    777
      THSTD=THSTD+ALINC                                          ORIG    778
      ALINC=(SEX(K)-DLN*SDX(K))**2+(SEY(K)-DLN*SDY(K))**2+       ORIG    779
     1(SEZ(K)-DLN*SDZ(K))**2                                     ORIG    780
      DMEN=DMEN+SQRT(ALINC)                                      ORIG    781
      DSTD=DSTD+ALINC                                            ORIG    782
      ALFX=(AOA*SA(K)+AOB)*SAX(K)                                ORIG    783
      ALFY=(AOA*SA(K)+AOB)*SAY(K)                                ORIG    784
      ALFZ=(AOA*SA(K)+AOB)*SAZ(K)                                ORIG    785
      ALINC=(SEX(K)-ALFX)**2+(SEY(K)-ALFY)**2+(SEZ(K)-ALFZ)**2   ORIG    786
      AOMN=AOMN+SQRT(ALINC)                                      ORIG    787
      AOSD=AOSD+ALINC                                            ORIG    788
      ALFX=HA*SAX(K)+HBX                                         ORIG    789
      ALFY=HA*SAY(K)+HBY                                         ORIG    790
      ALFZ=HA*SAZ(K)+HBZ                                         ORIG    791
      ALINC=(SEX(K)-ALFX)**2+(SEY(K)-ALFY)**2+(SEZ(K)-ALFZ)**2   ORIG    792
      HAMN=HAMN+SQRT(ALINC)                                      ORIG    793
      HASD=HASD+ALINC                                            ORIG    794
      SASM=SASM+SA(K)                                            ORIG    795
   17 CONTINUE                                                   ORIG    796
      FAC=1000.0                                                 ORIG    797
      ALMEN=ALMEN/XNDPM                                          ORIG    798
      ALSTD=SQRT((ALSTD-XNDPM*ALMEN**2)/(XNDPM-1.0))*FAC          ORIG    799
      DIMEN=DIMEN/XNDPM                                          ORIG    800
      DISTD=SQRT((DISTD-XNDPM*DIMEN**2)/(XNDPM-1.0))*FAC          ORIG    801
      AMEN=AMEN/XNDPM                                            ORIG    802
      BMEN=BMEN/XNDPM                                            ORIG    803
      CMEN=CMEN/XNDPM                                            ORIG    804
      DMEN=DMEN/XNDPM                                            ORIG    805
      ASTD=SQRT((ASTD-XNDPM*AMEN**2)/(XNDPM-1.0))*FAC            ORIG    806
      BSTD=SQRT((BSTD-XNDPM*BMEN**2)/(XNDPM-1.0))*FAC            ORIG    807
      CSTD=SQRT((CSTD-XNDPM*CMEN**2)/(XNDPM-1.0))*FAC            ORIG    808
      DSTD=SQRT((DSTD-XNDPM*DMEN**2)/(XNDPM-1.0))*FAC            ORIG    809
      AMEN=AMEN*FAC                                              ORIG    810
      BMEN=BMEN*FAC                                              ORIG    811
      CMEN=CMEN*FAC                                              ORIG    812
      DMEN=DMEN*FAC                                              ORIG    813
```

```
SUBROUTINE XMATH       74/74    OPT=1                    FTN 4.5+414        11/05/77  1 .12.37        PAGE    11

                 DIMEN=DIMEN*FAC                                                     ORIG      814
                 ALMEN=ALMEN*FAC                                                     ORIG      815
                 THMEN=THMEN/XNDPM                                                   ORIG      816
                 THSTD=SQRT((THSTD-XNDPM*THMEN**2)/(XNDPM-1.0))*FAC                  ORIG      817
  575            THMEN=THMEN*FAC                                                     ORIG      818
                 HAMN=HAMN/XNDPM                                                     ORIG      819
                 HASD=SQRT((HASD-XNDPM*HAMN**2)/(XNDPM-1.0))*FAC                     ORIG      820
                 HAMN=HAMN*FAC                                                       ORIG      821
                 AOMN=AOMN/XNDPM                                                     ORIG      822
  5 0            AOSD=SQRT((AOSD-XNDPM*AOMN**2)/(XNDPM-1.0))*FAC                     ORIG      823
                 AOMN=AOMN*FAC                                                       ORIG      824
                 SASMN=SASM*FAC/XNDPM                                                ORIG      825
                                                                                     ORIG      826
                 RETURN                                                              ORIG      827
  5              END                                                                 ORIG      828
```

```
      SUBROUTINE TRANS(EA,EB,EC,ED,PSI,PCH,ROL)              ORIG    829
      COMMON/TRAN/TR(3,3)                                    ORIG    830
C                                                            ORIG    831
      PI=3.141592654                                         ORIG    832
C                                                            ORIG    833
      DEN=SQRT(EA*EA+EB*EB+EC*EC+ED*ED)                      ORIG    834
      EA=EA/DEN                                              ORIG    835
      EB=EB/DEN                                              ORIG    836
      EC=EC/DEN                                              ORIG    837
      ED=ED/DEN                                              ORIG    838
C                                                            ORIG    839
      TR(1,1)=1.0-2.0*(EC*EC+ED*ED)                          ORIG    840
      TR(1,2)=2.0*(EB*EC-EA*ED)                              ORIG    841
      TR(1,3)=2.0*(EB*ED+EA*EC)                              ORIG    842
      TR(2,1)=2.0*(EB*EC+EA*ED)                              ORIG    843
      TR(2,2)=1.0-2.0*(EB*EB+ED*ED)                          ORIG    844
      TR(2,3)=2.0*(EC*ED-EA*EB)                              ORIG    845
      TR(3,1)=2.0*(EB*ED-EA*EC)                              ORIG    846
      TR(3,2)=2.0*(EC*ED+EA*EB)                              ORIG    847
      TR(3,3)=1.0-2.0*(EB*EB+EC*EC)                          ORIG    848
C                                                            ORIG    849
      PCH=ASIN(-TR(3,1))                                     ORIG    850
      IF(ABS(COS(PCH)).LE.0.0001)GO TO 2                     ORIG    851
    1 TEMP=(2.0*(EA*EA+EB*EB)-1.0)/COS(PCH)                  ORIG    852
      IF(TEMP.GT.1.0)TEMP=1.0                                ORIG    853
      IF(TEMP.LT.-1.0)TEMP=-1.0                              ORIG    854
      PSI=ACOS(TEMP)                                         ORIG    855
      IF(TR(2,1).LT.0.)PSI=2.0*PI-PSI                        ORIG    856
      TEMP=(2.0*(EA*EA+ED*ED)-1.0)/COS(PCH)                  ORIG    857
      IF(TEMP.GT.1.0)TEMP=1.0                                ORIG    858
      IF(TEMP.LT.-1.0)TEMP=-1.0                              ORIG    859
      ROL=ACOS(TEMP)                                         ORIG    860
      IF(TR(3,2).LT.0.0)ROL=-ROL                             ORIG    861
    2 RETURN                                                 ORIG    862
      END                                                    ORIG    863
```

```
SUBROUTINE XITOB       74/74    OPT=1                          FTN 4.5+414      11/05/77  15.12.37      PAGE    1

              SUBROUTINE XITOB(XB,YB,ZB,XIN,YIN,ZIN)                               ORIG       872
              COMMON/TEN/TR(3,3)                                                   ORIG       873
                                                                                   ORIG       874
              XB=TR(1,1)*XIN+TR(2,1)*YIN+TR(3,1)*ZIN                               ORIG       875
              YB=TR(1,2)*XIN+TR(2,2)*YIN+TR(3,2)*ZIN                               ORIG       876
              ZB=TR(1,3)*XIN+TR(2,3)*YIN+TR(3,3)*ZIN                               ORIG       877
              RETURN                                                               ORIG       878
              END                                                                  ORIG       879
```

```
SUBROUTINE BTOI        74/74    OPT=1                          FTN 4.5+414      11/05/77  15.12.37      PAGE    1

              SUBROUTINE BTOI(XB,YB,ZB,XIN,YIN,ZIN)                                ORIG       864
              COMMON/TEN/TR(3,3)                                                   ORIG       865
                                                                                   ORIG       866
              XIN=TR(1,1)*XB+TR(1,2)*YB+TR(1,3)*ZB                                 ORIG       867
              YIN=TR(2,1)*XB+TR(2,2)*YB+TR(2,3)*ZB                                 ORIG       868
              ZIN=TR(3,1)*XB+TR(3,2)*YB+TR(3,3)*ZB                                 ORIG       869
              RETURN                                                               ORIG       870
              END                                                                  ORIG       871
```

```
SUBROUTINE ATMOS       74/74    OPT=1                          FTN 4.5+414      11/05/77  15.12.37      PAGE    1

              SUBROUTINE ATMOS(PR,CA,ZA)                                           ORIG       880
        C*ALT,SPEED OF SOUND = FUNC OF PRESS (TO 36000 FT)                         ORIG       881
              ZA=-145241.3*(((PR/2116.2)**0.190284*75)-1.0)                        ORIG       882
              CA=49.0217**SQRT(518.69-.00357*ZA)                                   ORIG       883
              RETURN                                                               ORIG       884
              END                                                                  ORIG       885
```